

Enforcement of Q-Rung Orthopair Complex Fuzzy Set to Group Theoretical Approach

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ABSTRACT:

A complex fuzzy set is a vigorous framework to characterize novel machine learning algorithms. This set is more suitable and flexible compared to fuzzy sets, intuitionistic fuzzy sets, and bipolar fuzzy sets. In this article, we introduced the concept complex q-rung orthopair fuzzy subgroup which the extension of complex intuitionistic/Pythagorean fuzzy subgroups. Based on the complex q-rung orthopair fuzzy subgroup, several characterizations are discussed. Also, normality, cosets, homomorphism order of complex q-rung orthopair fuzzy subgroup were introduced and their properties were analyzed.

Keywords: fuzzy set, complex fuzzy set, fuzzy subgroup, additive group, q-rung complex fuzzy subgroup, coset, normality, homomorphism..

1.Introduction:

In 1965, Zadeh [1] presented the theory of fuzzy sets and discussed their initiatory results. The action of fuzzy set theory is a decisive structure to deal vagueness and uncertainty in real life problems. Thus, crisp sets commonly do not have suitable response and feedback for actual worldly conditions of happening issues. In addition, this particular set plays a remarkable role in various scientific fields with wide applications in topological spaces, medical diagnosis determination, coding theory, computer sciences, and module theory. The idea of fuzzy subgroups was introduced by Rosenfeld

[2] in 1971. The abstraction of fuzzy subgroups was proposed by Liu [3]. Later, these notions were discussed in [4,5,6]. Atanassov [7] initiated the theory of intuitionistic fuzzy sets and established the basic algebraic properties of intuitionistic fuzzy sets. The complex numbers with fuzzy sets were combined by Buckley [8]. Kim [9] presented the idea of the fuzzy order of elements of a group. Ajmal [10] established the fuzzy homomorphism theorems of fuzzy subgroups. He also discussed the fuzzy quotient group and correspondence theorem. Ray [11] initiated the notion of Cartesian product of fuzzy subgroup. Moreover, the volume and intricacy which exist in the

collected information of our daily life are developing rapidly with the phase shift of data. Then, there is regularly existing various sorts of uncertainty in that information is represented with complicated problems in different disciplines, such as biology, economics, social science, computer science, mathematics, and environmental science. With the development of science and technology, the decision making problems are becoming increasingly difficult. To overcome this drawback, Ramot et al. [12,13] presented the generalized form of fuzzy set by combining a phase term, called a complex fuzzy set. The efficiency of complex fuzzy logic in the respect of membership has a powerful role to deal with concrete problems. It is highly valuable for calculating unevenness, and also it is very useful way to address ambiguous ideas. Despite its efficacy, we have serious problems about physical features on complex membership related function.

Thus, it is highly essential to formulate extra theories of complicated fuzzy set relating intricate set members. This reasoning is a direct version of traditional fuzzy logic, which results in problem related fuzzy reasoning. Thus, it is not favorable for superficial membership function. This set has a very specific role in wide variety of applications in modern commanding systems especially those that forecast periodic events in which a number of variables are interconnected in complex ways and fuzzy operations cannot run it effectively. The fundamental set theoretic operations of complex fuzzy sets were presented by Zhang et al. [14]. Recently, the possible applications, which explain the novel ideas, including complex fuzzy sets in forecasting issues, solar activity, and time series were investigated by Thirunavukarasu et al. [15]. The complex fuzzy sets have wide applications in decision making, image restoration, and reasoning schemes. Ameri et al. [16] invented the of Engel fuzzy subgroups in 2015. The notion of complex vague soft sets were defined by Selvachandran [17]. Al-Husban and Salleh [18] developed a connection between complex fuzzy sets and group theory in 2016. Singh et al. [19] discussed the link between complex fuzzy set and metric spaces. In 2016,

Thirunavukarasu et al. [20] depicted the abstraction of complex fuzzy graph and find energy of this newly defined graph. In 2017, Alsarahead and Ahmed [21] presented a new abstraction of complex fuzzy subgroup. They also introduced novel conception complex fuzzy subring and complex fuzzy soft subgroups. These abstractions are completely different from Rosenfeld fuzzy subgroups [2] and Liu fuzzy subring [3]. The parabolic fuzzy subgroups were introduced by Makamba and Murali [24]. Then, certain algebraic properties of Engel fuzzy subgroups were discussed in [25]. Moreover, the Mohamadzahed et al. [26] established the novel concept of nilpotent fuzzy subgroup. The fuzzy homomorphism structures on fuzzy subgroups was discussed by Addis [27]. The algebraic structure between fuzzy sets and normed rings were proposed in [28]. Gulistan et al. [29] presented the notion of (α, β) -complex fuzzy hyper-ideal and investigated many algebraic properties of this phenomena. Liu and Shi [30] presented a novel framework to fuzzification of lattice, which is known as an M -hazy lattice. The complex fuzzy sub-algebra commenced in [31]. Currently, the novel environment of complex fuzzy set in decision making problems has been used frequently [32], owing to the existence of complex fuzzy information in several practical situations. Gulzar et al. [33] published a inventive theory of complex fuzzy subfields. The new development about $\mathcal{Q}$ -complex fuzzy subring was introduced in [34]. For other useful results of fuzzy subgroups not mentioned in the article, readers are referred to [35].

In 2013, Yager RR [36] invented Pythagorean Fuzzy Sets (PFSs). Most recently another amazing generalization of FSs is proposed by Yager RR [36], q -Rung Orthopair Fuzzy Sets (q -ROFSs) which models uncertain and incomplete information better than both IFSs and PFSs with high accuracy. q -ROFSs is more fruitful in many decision-making problems. In 2018. In 2019, Some results related to it were given by Peng X [37] and published Information measures for q -rung orthopair fuzzy sets. In certain situations, the PFSs unable to handle the problems like if membership degree $\eta = 0.8$ and non-membership

degree $\theta = 0.75$ then $\eta_2 + \theta_2 \geq 1.20251$ it is clear that PFSs fail to define this type of sets.. This concept is perfectly designed to represent imprecision and ambiguity in mathematical way and to produce a formalized tool to handle fuzziness to real problems. Pinar A et al. [38] suggested a q-rung orthopair fuzzy multi-criteria group decision making method for supplier selection based on a novel distance. In this article, we introduced the concept complex q-rung orthopair fuzzy subgroup which the extension of complex intuitionistic/Pythagorean fuzzy subgroups. Based on the complex q-rung orthopair fuzzy subgroup, several characterizations are discussed. Also, normality, cosets, homomorphism order of complex q-rung orthopair fuzzy subgroup were introduced and their properties were analyzed.

2. Preliminaries

In this section, we discuss basic definitions of Zadeh's fuzzy set, complex fuzzy set.

Definition-2.1: Let ' X ' be a crisp set. Then $J: X \rightarrow [0, 1]$ is called a fuzzy set, denoted by X . Hence $J(x)$ is called the degree of membership.

Rosenfeld was the first who worked on fuzzy subgroups and fuzzy graphs in (1971), (1975).

Definition-2.2: Let $J: X \rightarrow [0, 1]$ be a fuzzy subset of a group $(X, .)$. Then, J is said to be a fuzzy subgroup of $(X, .)$, if the following conditions hold,

- (i) $J(x.y) \geq \min\{J(x), J(y)\}$,
- (ii) $J(x^{-1}) \geq J(x)$, for all $x, y \in X$.

Ramot et.al, defined complex fuzzy set on classical set in 2002.

Definition-2.3: Let ' X ' be a crisp set and define J on X to be complex fuzzy set (CFS), where

$J = \{(x, J(x))/x \in X\}$ such that $M(x) = X \rightarrow \{\Delta_1/\Delta_1 \in \mathbb{C}, |\Delta_1| \leq 1\}$, provided that

$J(x) = p(x)e^{2\pi i\alpha(x)}$ where $p(x)$ and $\alpha(x) \in [0, 1]$.

Recently in 2021, Eman A. Abuhijleh, Mourad Massa'deh, Amani Sheimat, and Abdulazeez Alkouri are defined complex fuzzy subgroup.

Definition-2.4: Let $J(x) = X \rightarrow \{\Delta_1/\Delta_1 \in \mathbb{C}, |\Delta_1| \leq 1\}$ be a complex fuzzy subset of the group $(X, .)$. Then J is said to be a complex fuzzy subgroup (CFSG) of $(X, .)$, if the following conditions hold;

- (i) $J(x.y) \geq \min\{J(x), J(y)\}$,
- (ii) $J(x^{-1}) \geq J(x)$, for all $x, y \in X$.

which is equivalently, for each any $x, y \in X$

and

$J(x) = p(x)e^{2\pi i\alpha(x)}$, we get

- (i) $p(x.y) \geq \min\{p(x), p(y)\}$
- and
- $\alpha(x.y) \geq \min\{\alpha(x), \alpha(y)\}$
- (ii) $p(x^{-1}) \geq p(x)$ and $\alpha(x^{-1}) \geq \alpha(x)$.

In 2013, R.R.Yager established q-rung fuzzy set of classical set.

Definition-2.5: Let ' X ' be a classical set and defined ' Q ' on X to be q-rung orthopair fuzzy set, where $Q = \{(x, J(x), K(x))/x \in X\}$. Such that $J(x) \in [0, 1]$ and $K(x) \in [0, 1]$ are the degree of membership and non-membership of $x \in X$ which satisfying the condition

$$0 \leq J^q(x) + K^q(x) \leq 1, \text{ for all } x \in X.$$

Note that, in the previous definition if the condition was $0 \leq J(x) + K(x) \leq 1$, for all $x \in X$, then Q define an intuitionistic fuzzy set. $0 \leq J^2(x) + K^2(x) \leq 1$, called Pythagorean fuzzy set and $0 \leq J^3(x) + K^3(x) \leq 1$, called fermatean fuzzy set on X .

The following is the definition of q-rung orthopair fuzzy subgroup, by the 2020 study.

Definition-2.6: Let $(X, .)$ be a group and $Q = \{J, K\}$ be q-rung orthopair fuzzy set of X . Then Q is said to be q-rung orthopair fuzzy subgroup of X if the following conditions hold;

- (i) $J^q(x.y) \geq \min\{J^q(x), J^q(y)\}$ and
- $K^q(x.y) \leq \max\{K^q(x), K^q(y)\}$,

(ii) $J^q(x^{-1}) \geq J^q(x)$ and $K^q(x^{-1}) \leq K^q(x)$, $\forall x, y \in X$.

Note that, if $q = 1, 2, 3$ then Q is respectively called as intuitionistic, Pythagorean and fermatean fuzzy group.

The study developed the concept of complex q-rung orthopair fuzzy set (Q-rung CFS) in 2019.

Definition-2.7: Let 'X' be a classical set and defined 'A' on X be complex q-rung fuzzy set where $A = \{(x, J(x), K(x)) / x \in X\}$. Such that $J(x): X \rightarrow \{\Delta_1 / \Delta_1 \in \mathbb{C}, |\Delta_1| \leq 1\}$ and

$K(x): X \rightarrow \{\Delta_2 / \Delta_2 \in \mathbb{C}, |\Delta_2| \leq 1\}$ are the degree of membership and degree of non-membership of $x \in X$. More over $J(x) = \omega_1(x)e^{2\pi i \alpha(x)}$, $K(x) = \omega_2(x)e^{2\pi i \beta(x)}$

are satisfying the conditions $0 \leq \omega_1^q(x) + \omega_2^q(x) \leq 1$ and $0 \leq \alpha^q(x) + \beta^q(x) \leq 1$ for all $x \in U$. Then A defined q-rung complex fuzzy set on X.

In 2019, operations on q-rung complex fuzzy set, such as union, intersection and complement were defined.

Definition-2.8: Let $A_1 = \{J_1, K_1\}$ and $A_2 = \{J_2, K_2\}$ are q-rung complex fuzzy sets on X, where $J(x): X \rightarrow \{\omega_j(x)e^{2\pi i \alpha_j(x)} : 0 \leq \omega_j(x), \alpha_j(x) \leq 1\}$, and

$K(x): X \rightarrow \{\delta_j(x)e^{2\pi i \beta_j(x)} : 0 \leq \delta_j(x), \beta_j(x) \leq 1\}$.

(1) $A_1 \cap A_2 = \{J_1 \cap J_2, K_1 \cap K_2\}$, where

(a) $(J_1 \cap J_2)(x) = (\min\{\omega_1(x), \omega_2(x)\}e^{2\pi i \min\{\alpha_1(x), \alpha_2(x)\}})$

(b) $(K_1 \cap K_2)(x) = (\min\{\delta_1(x), \delta_2(x)\}e^{2\pi i \min\{\beta_1(x), \beta_2(x)\}})$

(2) $A_1 \cup A_2 = \{J_1 \cup J_2, K_1 \cup K_2\}$, where

(a) $(J_1 \cup J_2)(x) = (\max\{\omega_1(x), \omega_2(x)\}e^{2\pi i \max\{\alpha_1(x), \alpha_2(x)\}})$

(b) $(K_1 \cup K_2)(x) = (\max\{\delta_1(x), \delta_2(x)\}e^{2\pi i \max\{\beta_1(x), \beta_2(x)\}})$

Definition-2.9: Let $A = \{J, K\}$ be q-rung complex fuzzy set defined on X, where

$J(x): X \rightarrow \{\omega_1(x)e^{2\pi i \alpha(x)} : 0 \leq \omega_1(x), \alpha(x) \leq 1\}$, and

$K(x): X \rightarrow \{\omega_2(x)e^{2\pi i \beta(x)} : 0 \leq \omega_2(x), \beta(x) \leq 1\}$. Then the complement of 'A' defined by

$A^C = (K, J)$, where

(a) $J^C(x) = K = \omega_2(x)e^{2\pi i \beta(x)}$

(b) $K^C(x) = J = \omega_1(x)e^{2\pi i \alpha(x)}$

3. Q-rung complex fuzzy subgroups

A generalization of Pythagorean fuzzy subgroup and fermatean fuzzy subgroup is developed in the proceeding definition.

Definition-3.1: Let $(X, .)$ be a group and 'A' = $\{J = \omega_1 e^{2\pi i \alpha}, K = \omega_2 e^{2\pi i \beta}\}$ be a complex

q-rung fuzzy set of X. Then 'A' is said to be a complex q-rung fuzzy subgroup of X, where

$\omega_1^q + \omega_2^q \leq 1$ and $\alpha^q + \beta^q \leq 1$ if J and K have the following property,

$$\begin{aligned} 1(a) \quad J^q(x.y) &= \omega_1^q(x.y)e^{2\pi i \alpha^q(x.y)} \\ &\geq \min\{\omega_1^q(x)e^{2\pi i \alpha^q(x)}, \omega_1^q(y)e^{2\pi i \alpha^q(y)}\} \\ &= \min\{J^q(x), J^q(y)\} \end{aligned}$$

where $\omega_1^q(x.y) \geq \min\{\omega_1^q(x), \omega_1^q(y)\}$ and $\alpha^q(x.y) \geq \min\{\alpha^q(x), \alpha^q(y)\}$

$$\begin{aligned} 1(b) \quad K^q(x.y) &= \omega_2^q(x.y)e^{2\pi i \beta^q(x.y)} \\ &\leq \max\{\omega_2^q(x)e^{2\pi i \beta^q(x)}, \omega_2^q(y)e^{2\pi i \beta^q(y)}\} \\ &= \max\{K^q(x), K^q(y)\} \end{aligned}$$

where $\omega_2^q(x.y) \leq \max\{\omega_2^q(x), \omega_2^q(y)\}$ and $\beta^q(x.y) \leq \max\{\beta^q(x), \beta^q(y)\}$

$$2(a) \quad J^q(x^{-1}) = \omega_1^q(x^{-1}) \geq \omega_1^q(x)e^{2\pi i \alpha^q(x)}$$

where $\omega_1^q(x^{-1}) \geq \omega_1^q(x)$ and $\alpha^q(x^{-1}) \geq \alpha^q(x)$

$$2(b) \quad K^q(x^{-1}) = \omega_2^q(x^{-1})e^{2\pi i \beta^q(x^{-1})} \leq \omega_2^q(x)e^{2\pi i \beta^q(x)}$$

where $\omega_2^q(x^{-1}) \leq \omega_2^q(x)$ and $\beta^q(x^{-1}) \leq \beta^q(x)$

In previous definition,

if $q = 1$, then 'A' is called intuitionistic complex fuzzy group,

if $q = 2$, then 'A' is called pythagorean complex fuzzy group,

if $q = 1$, then 'A' is called fermatean complex fuzzy group.

Proposition-3.2: Let 'A' be a complex q-rung fuzzy subgroup of a group $(X, .)$. Then the following holds;

$$1. \quad J^q(e) = \omega_1^q(e)e^{2\pi i \alpha^q(e)} \geq \omega_1^q(x)e^{2\pi i \alpha^q(x)} = J^q(x)$$

where $\omega_1^q(e) \geq \omega_1^q(x)$ and $\alpha^q(e) \geq \alpha^q(x)$.

$$2. \quad K^q(e) = \omega_2^q(e)e^{2\pi i \beta^q(e)} \leq \omega_2^q(x)e^{2\pi i \beta^q(x)} = K^q(x)$$

where $\omega_2^q(e) \leq \omega_2^q(x)$ and $\beta^q(e) \leq \beta^q(x)$.

$$3. \quad J^q(x^{-1}) = \omega_1^q(x^{-1})e^{2\pi i \alpha^q(x^{-1})} = \omega_1^q(x)e^{2\pi i \alpha^q(x)} = J^q(x)$$

where $\omega_1^q(x^{-1}) = \omega_1^q(x)$ and $\alpha^q(x^{-1}) = \alpha^q(x)$.

$$4. \quad K^q(x^{-1}) = \omega_2^q(x^{-1})e^{2\pi i \beta^q(x^{-1})} = \omega_2^q(x)e^{2\pi i \beta^q(x)} = K^q(x)$$

where $\omega_2^q(x^{-1}) = \omega_2^q(x)$ and $\beta^q(x^{-1}) = \beta^q(x)$.

for all $x \in X$, where 'e' is the identity of all elements.

Proof: Since 'A' is q-rung fuzzy subgroup of a group, then by definition-3.1, '1' and '2' can be showed as follow;

$$J^q(e) = \omega_1^q(e)e^{2\pi i \alpha^q(e)}$$

$$\begin{aligned} J^q(e) &= \omega_1^q(x.x^{-1})e^{2\pi i \alpha^q(x.x^{-1})} \\ &\geq \min\{\omega_1^q(x)e^{2\pi i \alpha^q(x)}, \omega_1^q(x^{-1})e^{2\pi i \alpha^q(x^{-1})}\} \\ &\geq \min\{\omega_1^q(x), \omega_1^q(x^{-1})\}e^{2\pi i \min\{\alpha^q(x), \alpha^q(x^{-1})\}} \\ &\geq \omega_1^q(x)e^{2\pi i \alpha^q(x)} \geq J^q(x). \end{aligned}$$

In addition,

$$K^q(e) = \omega_2^q(e)e^{2\pi i \beta^q(e)}$$

$$\begin{aligned} K^q(e) &= \omega_2^q(x.x^{-1})e^{2\pi i \beta^q(x.x^{-1})} \\ &\leq \max\{\omega_2^q(x)e^{2\pi i \beta^q(x)}, \omega_2^q(x^{-1})e^{2\pi i \beta^q(x^{-1})}\} \\ &\leq \max\{\omega_2^q(x), \omega_2^q(x^{-1})\}e^{2\pi i \min\{\beta^q(x), \beta^q(x^{-1})\}} \\ &\leq \omega_2^q(x)e^{2\pi i \beta^q(x)} \leq K^q(x). \end{aligned}$$

Similarly, '3' and '4' can be proved.

In the following result, we exhibit that any complex intuitionistic fuzzy subgroup/Pythagorean fuzzy subgroup is complex q-rung fuzzy subgroup, whereas complex intuitionistic/Pythagorean is a subclass of complex q-rung fuzzy set.

Theorem-3.3: If 'A' is a complex intuitionistic fuzzy subgroup of the group $(X, .)$, then 'A' is a complex q-rung orthopair fuzzy subgroup of the group $(X, .)$.

Proof: At first, to claim that

$$\begin{aligned} \omega_1^q(x.y)e^{2\pi i \alpha^q(x.y)} &\geq \min\{\omega_1^q(x)e^{2\pi i \alpha^q(x)}, \omega_1^q(y)e^{2\pi i \alpha^q(y)}\} \text{ and} \\ \omega_2^q(x.y)e^{2\pi i \beta^q(x.y)} &\leq \max\{\omega_2^q(x)e^{2\pi i \beta^q(x)}, \omega_2^q(y)e^{2\pi i \beta^q(y)}\}. \end{aligned}$$

We know that, 'A' is a complex intuitionistic fuzzy subgroup, then

$$\begin{aligned} \omega_1(x.y)e^{2\pi i \alpha(x.y)} &\geq \min\{\omega_1(x)e^{2\pi i \alpha(x)}, \omega_1(y)e^{2\pi i \alpha(y)}\} \text{ and} \\ \omega_2(x.y)e^{2\pi i \beta(x.y)} &\leq \max\{\omega_2(x)e^{2\pi i \beta(x)}, \omega_2(y)e^{2\pi i \beta(y)}\} \end{aligned}$$

where $\omega_1 + \omega_2 \leq 1$ and $\alpha + \beta \leq 1$.

Then we consider the following four cases.

Case-(i): Let

$$\begin{aligned} \omega_1(x)e^{2\pi i \alpha(x)} &\geq \omega_1(y)e^{2\pi i \alpha(y)} \quad \text{and} \\ \omega_2(x)e^{2\pi i \beta(x)} &\geq \omega_2(y)e^{2\pi i \beta(y)}, \text{ then} \\ \omega_1(x.y)e^{2\pi i \alpha(x.y)} &\geq \omega_1(y)e^{2\pi i \alpha(y)}. \end{aligned}$$

Now, consider

$$\omega_1^q(x.y)e^{2\pi i\alpha^q(x.y)} \geq \omega_1^q(y)e^{2\pi i\alpha^q(y)} \geq \min\{\omega_1^q(x)e^{2\pi i\alpha^q(x)}, \omega_1^q(y)e^{2\pi i\alpha^q(y)}\}.$$

Moreover,

$$\omega_2^q(x.y)e^{2\pi i\beta^q(x.y)} \leq \omega_2^q(y)e^{2\pi i\beta^q(y)}$$

Now, consider

$$\omega_2^q(x.y)e^{2\pi i\beta^q(x.y)} \leq \omega_2^q(y)e^{2\pi i\beta^q(y)} \leq \max\{\omega_2^q(x)e^{2\pi i\beta^q(x)}, \omega_2^q(y)e^{2\pi i\beta^q(y)}\}$$

Case-(iii): Let

$$\omega_1(x)e^{2\pi i\alpha(x)} \leq \omega_1(y)e^{2\pi i\alpha(y)} \quad \text{and}$$

$$\omega_2(x)e^{2\pi i\beta(x)} \geq \omega_2(y)e^{2\pi i\beta(y)}, \text{ then}$$

$$\omega_1(x.y)e^{2\pi i\alpha(x.y)} \geq \omega_1(x)e^{2\pi i\alpha(x)}.$$

Now, consider

$$\omega_1^q(x.y)e^{2\pi i\alpha^q(x.y)} \geq \omega_1^q(x)e^{2\pi i\alpha^q(x)} \geq \min\{\omega_1^q(x)e^{2\pi i\alpha^q(x)}, \omega_1^q(y)e^{2\pi i\alpha^q(y)}\}.$$

Moreover,

$$\omega_2^q(x.y)e^{2\pi i\beta^q(x.y)} \leq \omega_2^q(y)e^{2\pi i\beta^q(y)}$$

Now, consider

$$\omega_2^q(x.y)e^{2\pi i\beta^q(x.y)} \leq \omega_2^q(x)e^{2\pi i\beta^q(x)} \leq \max\{\omega_2^q(x)e^{2\pi i\beta^q(x)}, \omega_2^q(y)e^{2\pi i\beta^q(y)}\}.$$

Case-(iv): Let

$$\omega_1(x)e^{2\pi i\alpha(x)} \geq \omega_1(y)e^{2\pi i\alpha(y)} \quad \text{and}$$

$$\omega_2(x)e^{2\pi i\beta(x)} \leq \omega_2(y)e^{2\pi i\beta(y)}, \text{ then with same argument of case-(iii), we get the result.}$$

Case-(ii): Let

$$\omega_1(x)e^{2\pi i\alpha(x)} \leq \omega_1(y)e^{2\pi i\alpha(y)} \quad \text{and}$$

$$\omega_2(x)e^{2\pi i\beta(x)} \leq \omega_2(y)e^{2\pi i\beta(y)}, \text{ then with same argument of case-(i), we get the result.}$$

Secondly, since

$$\omega_1(x^{-1})e^{2\pi i\alpha(x^{-1})} \geq \omega_1(x)e^{2\pi i\alpha(x)} \quad \text{and}$$

$$\omega_2(x^{-1})e^{2\pi i\beta(x^{-1})} \leq \omega_2(y)e^{2\pi i\beta(y)}, \text{ then}$$

$$\omega_1^q(x^{-1})e^{2\pi i\alpha^q(x^{-1})} \geq \omega_1^q(x)e^{2\pi i\alpha^q(x)} \quad \text{and}$$

$$\omega_2^q(x^{-1})e^{2\pi i\beta^q(x^{-1})} \leq \omega_2^q(y)e^{2\pi i\beta^q(y)} \text{ too.}$$

The converse of the above theorem-3.3 is not always true. The following example shown the proof.

Example-3.4: For the set $X = \{1, -1, i, -i\}$, define a group $(X, .)$, where $'.'$ is the known multiplication. Also define $A = \{J, K\}$ be a complex q-rung fuzzy set on U , where

$$J(1) = 0.6e^{2\pi i(0.7)} \quad K(1) = 0.7e^{2\pi i(0.8)}$$

$$J(-1) = 0.7e^{2\pi i(0.4)} \quad K(-1) = 0.6e^{2\pi i(0.5)}$$

$$J(i) = 0.3e^{2\pi i(0.4)} \quad K(i) = 0.4e^{2\pi i(0.2)}$$

$$J(-i) = 0.3e^{2\pi i(0.4)} \quad K(-i) = 0.4e^{2\pi i(0.2)}$$

Now to check that definition-2.4 is satisfied. We get for $x = 1, q = 2$ that

$$(0.6)^2 + (0.7)^2 < 1 \quad \text{and} \quad (0.7)^2 + (0.8)^2 < 1,$$

both are in the closed interval $[0, 1]$. It is easy to check that for all $x \in X$, the property satisfied and 'A' is complex q-rung fuzzy set. But it not complex intuitionistic fuzzy set where at $\omega = -1$, we get $0.7+0.6 \not\leq 1$. In addition, this set

$A = \{J, K\}$ is complex q-rung fuzzy subgroup, see definition-3.1.

Proposition-3.5: For complex q-rung fuzzy set 'A' of a group $(X, .)$, it is a complex q-rung fuzzy subgroup if and only if

$$(1) \quad J^q(x.y^{-1}) = \omega_1^q(x.y^{-1})e^{2\pi i\alpha^q(x.y^{-1})}$$

$$\geq \min\{\omega_1^q(x)e^{2\pi i\alpha^q(x)}, \omega_1^q(y)e^{2\pi i\alpha^q(y)}\}$$

$$= \min\{J^q(x), J^q(y)\}, \text{ where}$$

$$\omega_1^q(x.y^{-1}) \geq \min\{\omega_1^q(x), \omega_1^q(y)\} \quad \text{and}$$

$$\alpha^q(x.y^{-1}) \geq \min\{\alpha^q(x), \alpha^q(y)\}.$$

$$(2) \quad K^q(x.y^{-1}) = \omega_2^q(x.y^{-1})e^{2\pi i\beta^q(x.y^{-1})}$$

$$\leq \max\{\omega_2^q(x)e^{2\pi i\beta^q(x)}, \omega_2^q(y)e^{2\pi i\beta^q(y)}\}$$

$$= \max\{K^q(x), K^q(y)\}, \text{ where}$$

$$\omega_2^q(x.y^{-1}) \leq \max\{\omega_2^q(x), \omega_2^q(y)\} \quad \text{and}$$

$$\beta^q(x.y^{-1}) \leq \max\{\beta^q(x), \beta^q(y)\}.$$

Proof: The condition is necessary.

According to proposition-3.2, we have

$$\omega_1^q(x^{-1})e^{2\pi i\alpha^q(x^{-1})} = \omega_1^q(x)e^{2\pi i\alpha^q(x)} \quad \text{and} \\ \omega_2^q(x^{-1})e^{2\pi i\beta^q(x^{-1})} = \omega_2^q(x)e^{2\pi i\beta^q(x)}$$

for all $x \in X$, then results follow by definition-3.1.

The condition is sufficient.

At first, A is complex q-rung fuzzy set and is defined on group $(X, .)$, then

$$(a1) J^q(e) = \omega_1^q(e)e^{2\pi i\alpha^q(e)} \\ = \omega_1^q(x.x^{-1})e^{2\pi i\alpha^q(x.x^{-1})}$$

$$\geq \omega_1^q(x)e^{2\pi i\alpha^q(x)} = J^q(x)$$

where $\omega_1^q(x.x^{-1}) \geq \omega_1^q(x)$ and $\alpha^q(x.x^{-1}) \geq \alpha^q(x)$

$$(b1) J^q(x^{-1}) = \omega_1^q(x^{-1})e^{2\pi i\alpha^q(x^{-1})} \\ = \omega_1^q(e.x^{-1})e^{2\pi i\alpha^q(e.x^{-1})} \\ \geq \min\{\omega_1^q(e)e^{2\pi i\alpha^q(e)}, \omega_1^q(x)e^{2\pi i\alpha^q(x)}\} \\ = \min\{\omega_1^q(e), \omega_1^q(x)\}e^{2\pi i\min\{\alpha^q(e), \alpha^q(x)\}}$$

$$= \omega_1^q(x)e^{2\pi i\alpha^q(x)} = J^q(x) \text{ by (a1).}$$

$$(c1) J^q(x.y) = J^q(x.(y^{-1})^{-1}) \\ \geq \min\{J^q(x), J^q(y^{-1})\}$$

$$\geq \min\{J^q(x), J^q(y)\} \text{ by (b1).}$$

Similarly, we have

$$(a2) K^q(e) = \omega_2^q(e)e^{2\pi i\beta^q(e)} \\ = \omega_2^q(x.x^{-1})e^{2\pi i\beta^q(x.x^{-1})} \leq \omega_2^q(x)e^{2\pi i\beta^q(x)} \\ = J^q(x)$$

where $\omega_2^q(x.y^{-1}) \leq \omega_2^q(x)$ and $\beta^q(x.y^{-1}) \leq \beta^q(x)$

$$(b2) K^q(x^{-1}) = \omega_2^q(x^{-1})e^{2\pi i\beta^q(x^{-1})} \\ = \omega_2^q(e.x^{-1})e^{2\pi i\beta^q(e.x^{-1})} \\ \leq \max\{\omega_2^q(e)e^{2\pi i\beta^q(e)}, \omega_2^q(x)e^{2\pi i\beta^q(x)}\} \\ = \max\{\omega_2^q(e), \omega_2^q(x)\}e^{2\pi i\min\{\beta^q(e), \beta^q(x)\}} \\ = \omega_2^q(x)e^{2\pi i\beta^q(x)} = K^q(x) \text{ by (a2).}$$

$$(c2) K^q(x.y) = K^q(x.(y^{-1})^{-1}) \\ \leq \max\{K^q(x), K^q(y^{-1})\} \leq \\ \max\{K^q(x), K^q(y)\} \text{ by (b2).}$$

Finally, by (c1) and (c2) the first condition is satisfied and by (b1) and (b2) the second condition is satisfied.

Hence 'A' is complex q-rung orthopair fuzzy subgroup of a group $(X, .)$.

Proposition-3.6: If 'A' and 'B' be two complex q-rung orthopair fuzzy subgroup of a group $(X, .)$, then $A \cap B$ is complex q-rung orthopair fuzzy subgroup of a group $(X, .)$.

Proof:

Since 'A' and 'B' be two complex q-rung orthopair fuzzy subgroup of a group $(X, .)$ and using proposition-3.5, then

$$(i) J_{A \cap B}^q(x.y^{-1}) = \omega_{A \cap B}^q(x.y^{-1})e^{2\pi i\alpha_{A \cap B}^q(x.y^{-1})} \\ = \min\left\{\omega_A^q(x.y^{-1}), \omega_B^q(x.y^{-1})\right\} e^{2\pi i\min\left\{\alpha_A^q(x.y^{-1}), \alpha_B^q(x.y^{-1})\right\}} \\ \geq \min\left\{\min\{\omega_A^q(x), \omega_B^q(x)\}, \min\{\omega_A^q(y), \omega_B^q(y)\}\right\} \\ e^{2\pi i\{\min\{\alpha_A^q(x), \alpha_A^q(y)\}, \min\{\alpha_B^q(x), \alpha_B^q(y)\}\}} \\ = \min\left\{\min\{\omega_{A \cap B}^q(x), \omega_{A \cap B}^q(x)\}, \min\{\omega_{A \cap B}^q(y), \omega_{A \cap B}^q(y)\}\right\} \\ = \min\{J_{A \cap B}^q(x), J_{A \cap B}^q(y)\} \\ (ii) K_{A \cap B}^q(x.y^{-1}) = \omega_{A \cap B}^q(x.y^{-1})e^{2\pi i\beta_{A \cap B}^q(x.y^{-1})} \\ = \max\left\{\omega_A^q(x.y^{-1}), \omega_B^q(x.y^{-1})\right\} e^{2\pi i\min\left\{\beta_A^q(x.y^{-1}), \beta_B^q(x.y^{-1})\right\}} \leq \\ \min\left\{\min\{\omega_A^q(x), \omega_B^q(x)\}, \min\{\omega_A^q(y), \omega_B^q(y)\}\right\} \\ e^{2\pi i\{\min\{\beta_A^q(x), \beta_A^q(y)\}, \min\{\beta_B^q(x), \beta_B^q(y)\}\}} \\ = \min\left\{\min\left\{\omega_{A \cap B}^q(x), \omega_{A \cap B}^q(x)\right\}, \min\left\{\omega_{A \cap B}^q(y), \omega_{A \cap B}^q(y)\right\}\right\} \\ = \min\{K_{A \cap B}^q(x), K_{A \cap B}^q(y)\}$$

The proof is completed.

Example-3.7: Let $(X, \cdot) = (Z, +)$ be a group also $A_1 = 3Z$ and $A_2 = 2Z$ be two complex q-rung orthopair fuzzy subgroup of Z , where

$$A_j = \left\{ J_{A_j} = \omega_{1A_j} e^{2\pi i \alpha_{A_j}(x)}, K_{A_j} = \omega_{2A_j} e^{2\pi i \beta_{A_j}(x)} \right\}; j = 1, 2$$

They defined by

$$J_{A_1}(x) = \begin{cases} 0.4e^{2\pi i(0.5)}, & x \in 3Z \\ 0.0e^{2\pi i(0.0)}, & \text{otherwise} \end{cases}$$

$$K_{A_1}(x) = \begin{cases} 0.0e^{2\pi i(0.0)}, & x \in 3Z \\ 0.5e^{2\pi i(0.5)}, & \text{otherwise} \end{cases}$$

$$J_{A_2}(x) = \begin{cases} 0.04e^{2\pi i(0.1)}, & x \in 2Z \\ 0.0e^{2\pi i(0.0)}, & \text{otherwise} \end{cases}$$

$$K_{A_2}(x) = \begin{cases} 0.3e^{2\pi i(0.2)}, & x \in 2Z \\ 0.4e^{2\pi i(0.3)}, & \text{otherwise} \end{cases}$$

For $A = A_1 \cup A_2$, we get

$$J_A(x) = \begin{cases} 0.4e^{2\pi i(0.5)}; & x \in 3Z \\ 0.4e^{2\pi i(0.1)}; & x \in 2Z - 3Z \\ 0.0e^{2\pi i(0.0)}; & \text{otherwise} \end{cases}$$

$$K_A(x) = \begin{cases} 0.0e^{2\pi i(0.0)}; & x \in 3Z \\ 0.03e^{2\pi i(0.2)}; & x \in 2Z - 3Z \\ 0.4e^{2\pi i(0.3)}; & \text{otherwise} \end{cases}$$

Here, $x_1 = 9$ and $x_2 = -2$, then

$$J^q(9 + (-2)) = J^q(7) = 0.0e^{2\pi i(0.0)} \text{ and}$$

$$\min\{J^q(9), J^q(-2)\}$$

$$= \min\{(0.4)^2 e^{2\pi i(0.5)^2}, (0.04)^2 e^{2\pi i(0.1)^2}\}$$

$$= \min\{0.16, 0.0016\} e^{2\pi i \min\{0.25, 0.01\}}$$

$$= 0.0016 e^{2\pi i(0.01)}$$

Hence, $0.0e^{2\pi i(0.0)} \not\geq 0.0016 e^{2\pi i(0.01)}$.

$$(i. e) J^q(9 + (-2)) \not\geq \min\{J^q(9), J^q(-2)\}.$$

Similarly, for non-membership, we have (after calculation)

$$K^q(9 + (-2)) \not\leq \max\{K^q(9), K^q(-2)\}, \text{ where}$$

$$0.16e^{2\pi i(0.09)} \not\leq 0.09e^{2\pi i(0.04)}.$$

Then $A = A_1 \cup A_2$ is not a complex q-rung orthopair fuzzy subgroup of $(Z, +)$.

Proposition-3.8: For a complex q-rung fuzzy set $A = \{J = \omega_1 e^{2\pi i \alpha}, K = \omega_2 e^{2\pi i \beta}\}$ of a group $(X, \cdot)$. Then

$$J^q(x.x.x \dots x) = \omega_1^q(x.x.x \dots x) e^{2\pi i \alpha^q(x.x.x \dots x)} \geq \omega_1^q(x) e^{2\pi i \alpha^q(x)} = J^q(x), \text{ where}$$

$$\omega_1^q(x.x.x \dots x) \geq \omega_1^q(x) \quad \text{and} \quad \alpha^q(x.x.x \dots x) \geq \alpha^q(x).$$

Also,

$$K^q(x.x.x \dots x) = \omega_2^q(x.x.x \dots x) e^{2\pi i \beta^q(x.x.x \dots x)} \leq \omega_2^q(x) e^{2\pi i \beta^q(x)} = K^q(x), \text{ where}$$

$$\omega_2^q(x.x.x \dots x) \leq \omega_2^q(x) \quad \text{and} \quad \beta^q(x.x.x \dots x) \leq \beta^q(x).$$

Proof: We can prove this proposition by induction results.

Now, $\omega_1^q(x.x) e^{2\pi i \alpha^q(x.x)} \geq \omega_1^q(x) e^{2\pi i \alpha^q(x)}$, where

$$\omega_1^q(x.x) \geq \omega_1^q(x) \text{ and } \alpha^q(x.x) \geq \alpha^q(x).$$

Also, $\omega_2^q(x.x) e^{2\pi i \beta^q(x.x)} \leq \omega_2^q(x) e^{2\pi i \beta^q(x)}$, where

$$\omega_2^q(x.x) \leq \omega_2^q(x) \text{ and } \beta^q(x.x) \leq \beta^q(x).$$

Hence the proof.

Theorem-3.9: For a complex q-rung orthopair fuzzy set $A = \{J = \omega_1 e^{2\pi i \alpha}, K = \omega_2 e^{2\pi i \beta}\}$ of a group $(X, \cdot)$. The set $S = \{x \in X / \omega_1^q(e) e^{2\pi i \alpha^q(e)} = \omega_1^q(x) e^{2\pi i \alpha^q(x)} \text{ and } \omega_2^q(e) e^{2\pi i \beta^q(e)} = \omega_2^q(x) e^{2\pi i \beta^q(x)}\}$ is a subgroup of X , where 'e' is the identity element.

Proof: First, we have $e \in S$, hence 'S' is not empty.

Moreover, we need to show that $x.y^{-1} \in S$ for all $x, y \in S$.

Assume that $x, y \in S$, where A is a complex q-rung orthopair fuzzy subgroup of X , then by proposition-3.8,

$$\begin{aligned} & \omega_l^q(x.y^{-1})e^{2\pi i\alpha^q(xy^{-1})} \\ & \geq \min\{\omega_l^q(x)e^{2\pi i\alpha^q(x)}, \omega_l^q(y)e^{2\pi i\alpha^q(y)}\} \\ & = \omega_l^q(e)e^{2\pi i\alpha^q(e)}, \text{ according to definition of } S. \end{aligned}$$

But, $x.y^{-1} \in A$, hence

$$\omega_l^q(e)e^{2\pi i\alpha^q(e)} \geq \omega_l^q(x.y^{-1})e^{2\pi i\alpha^q(xy^{-1})}, \text{ by proposition-3.2, so that}$$

$$\text{equality holds and } \omega_l^q(e)e^{2\pi i\alpha^q(e)} = \omega_l^q(x.y^{-1})e^{2\pi i\alpha^q(xy^{-1})}.$$

Similarly, we can show $\omega_2^q(e)e^{2\pi i\beta^q(e)} = \omega_2^q(x.y^{-1})e^{2\pi i\beta^q(xy^{-1})}$, by proposition-3.2 and proposition-3.8. So that, $x.y^{-1} \in S$ and S is a subgroup of X .

4. Complex q-rung orthopair fuzzy coset

In this section, we express complex q-rung orthopair fuzzy left coset and right coset and gives equivalent conditions and some properties for it.

Definition-4.1: Let $A = (J = \omega_1 e^{2\pi i\alpha}, K = \omega_2 e^{2\pi i\beta})$ be a complex q-rung orthopair fuzzy subgroup of a group $(X, .)$. Then for $a \in X$, the complex q-rung orthopair fuzzy left coset of A is the complex q-rung orthopair fuzzy set

$$aA = \{aJ = a\omega_1 e^{2\pi i\alpha a}, K = a\omega_2 e^{2\pi i\beta a}\},$$

which defined for membership by,

$$\begin{aligned} (aJ)^q(x) &= a\omega_1^q(x)e^{2\pi i\alpha^q(ax)} = \\ \omega_1^q(a^{-1}x)e^{2\pi i\alpha^q(a^{-1}x)} &= J^q(a^{-1}x). \end{aligned}$$

Also for non-membership is defined by,

$$\begin{aligned} (aK)^q(x) &= a\omega_2^q(x)e^{2\pi i\beta^q(ax)} = \\ \omega_2^q(a^{-1}x)e^{2\pi i\beta^q(a^{-1}x)} &= K^q(a^{-1}x). \end{aligned}$$

In the similar way, the complex q-rung orthopair fuzzy right coset of A is the complex q-rung orthopair fuzzy set $Aa = (Ja, Ka)$ and defined by

$$(Ja)^q(x) = J^q(x.a^{-1}) \quad \text{and} \quad (Ka)^q(x) = K^q(x.a^{-1})$$

for membership and non-membership respectively.

Definition-4.2: Let $A = (J(x) = \omega_1(x)e^{2\pi i\alpha(x)}, K(x) = \omega_2(x)e^{2\pi i\beta(x)})$ be a complex q-rung orthopair fuzzy subgroup of G of a group $(X, .)$. Then ' A ' is a complex q-rung orthopair fuzzy normal subgroup of $(X, .)$ if every complex q-rung orthopair fuzzy left coset is a complex q-rung orthopair fuzzy right coset of A in X . Equivalently $aA = Aa$.

Definition-4.3: Let $(X, .) = (Z_5, +_5)$ be a additive group with modulo integer 5. Define a complex q-rung orthopair fuzzy set $A = (J(x) = \omega_1(x)e^{2\pi i\alpha(x)}, K(x) = \omega_2(x)e^{2\pi i\beta(x)})$ as follows

$$J(x) = \begin{cases} 0.8e^{2\pi i(0.5)}, & x = 0 \\ 0.7e^{2\pi i(0.6)}, & x = 1 \\ 0.7e^{2\pi i(0.8)}, & x = 2 \\ 0.2e^{2\pi i(0.5)}, & x = 3 \\ 0.9e^{2\pi i(0.8)}, & x = 4 \end{cases} \quad \text{and}$$

$$K(x) = \begin{cases} 0.1e^{2\pi i(0.3)}, & x = 0 \\ 0.6e^{2\pi i(0.5)}, & x = 1 \\ 0.5e^{2\pi i(0.4)}, & x = 2 \\ 0.5e^{2\pi i(0.7)}, & x = 3 \\ 0.3e^{2\pi i(0.3)}, & x = 4 \end{cases}$$

Now, we need to show that ' A ' is a complex q-rung orthopair fuzzy subgroup of X .

For $x = 1 \in X$, the complex q-rung orthopair fuzzy left coset of A is, the complex q-rung orthopair fuzzy set $1A = (1J, 1K)$ and defined by

$$\begin{aligned} (1J)^q(x) &= J^q(1^{-1} +_5 x) = \\ \omega_1^q(1^{-1} +_5 x)e^{2\pi i\alpha^q(1^{-1} +_5 x)} &\text{ and} \end{aligned}$$

$$\begin{aligned} (1K)^q(x) &= K^q(1^{-1} +_5 x) \\ &= \omega_2^q(1^{-1} +_5 x)e^{2\pi i\beta^q(1^{-1} +_5 x)}. \end{aligned}$$

Similarly, the complex q-rung orthopair fuzzy right coset of A is, the complex q-rung orthopair fuzzy set $A1 = (J1, K1)$ and defined by

$$\begin{aligned} (J1)^q(x) &= J^q(x +_5 1^{-1}) = \\ \omega_1^q(x +_5 1^{-1})e^{2\pi i\alpha^q(x +_5 1^{-1})} &\text{ and} \end{aligned}$$

$$\begin{aligned} (K1)^q(x) &= K^q(x +_5 1^{-1}) \\ &= \omega_2^q(x +_5 1^{-1})e^{2\pi i\beta^q(x +_5 1^{-1})}. \end{aligned}$$

Let us assume that $x = 0$, then

$$\begin{aligned}
(1J)^q(0) &= J^q(1^{-1} +_5 0) \\
&= \omega_1^q(1^{-1} +_5 0)e^{2\pi i \alpha^q(1^{-1} +_5 0)} \\
&= \omega_1^q(4 +_5 0)e^{2\pi i \alpha^q(4 +_5 0)} \\
&= \omega_1^q(4)e^{2\pi i \alpha^q(4)} \\
&= 0.9e^{2\pi i(0.8)} \quad \text{and} \\
(1K)^q(0) &= K^q(1^{-1} +_5 0) \\
&= \omega_2^q(1^{-1} +_5 0)e^{2\pi i \beta^q(1^{-1} +_5 0)} \\
&= \omega_2^q(4 +_5 0)e^{2\pi i \beta^q(4 +_5 0)} \\
&= \omega_2^q(4)e^{2\pi i \beta^q(4)} \\
&= 0.3e^{2\pi i(0.3)}
\end{aligned}$$

In addition, we will show, $(1J)^q(0) = 0.9e^{2\pi i(0.8)}$ and $(1K)^q(0) = 0.3e^{2\pi i(0.3)}$.

Hence, $(1J)^q(0) = (J1)^q(0)$ and $(1K)^q(0) = (K1)^q(0)$.

For $x = 1, 2, 3, 4$ it is easy to verify that $(1J)^q(x) = (J1)^q(x)$ and $(1K)^q(x) = (K1)^q(x)$.

Then, $1.A = A.1$.

In the similar way we can show that $xA = Ax$ for all $x \in X$.

Hence, 'A' is a complex q-rung orthopair fuzzy normal subgroup of $(Z_5, +_5)$.

Proposition-4.4: For a group $(X, .)$ that is defined on complex q-rung orthopair fuzzy subgroup, 'A'. Then, 'A' is complex q-rung orthopair fuzzy normal subgroup of X if and only if $(J)^q(x) = J^q(a.x.a^{-1})$ and $(K)^q(x) = K^q(a.x.a^{-1})$, for all $a, x \in X$.

Proof: We first consider,

$$\begin{aligned}
J^q(x) &= \omega_1^q(x)e^{2\pi i \alpha^q(x)} \\
&= \omega_1^q(x.e)e^{2\pi i \alpha^q(x.e)} \\
&= \omega_1^q((x.a).a^{-1})e^{2\pi i \alpha^q((x.a).a^{-1})} \\
&= \omega_1^q(a^{-1}.(x.a))e^{2\pi i \alpha^q(a^{-1}.(x.a))} \\
&= J^q(a^{-1}.(x.a))
\end{aligned}$$

where 'A' is complex q-rung orthopair fuzzy normal subgroup of X .

But $a = (a^{-1})^{-1}$ and by similarly $J^q(x) = J^q(a.x.a^{-1})$.

Also, it is simply to check that $K^q(x) = K^q(a.x.a^{-1})$ too.

Conversely,

$$\begin{aligned}
J^q(a.x) &= \omega_1^q(a.x)e^{2\pi i \alpha^q(a.x)} \\
&= \omega_1^q(a.x.e)e^{2\pi i \alpha^q(a.x.e)} \\
&= \omega_1^q(a.(x.a).a^{-1})e^{2\pi i \alpha^q(a.(x.a).a^{-1})} \\
&= J^q(x.a).
\end{aligned}$$

Also, it is easy to claim that, $K^q(a.x) = K^q(x.a)$.

Therefore, we obviously, 'A' is complex q-rung orthopair fuzzy subgroup of X .

Theorem-4.5: Let 'A' be a complex q-rung orthopair fuzzy normal subgroup of a group $(X, .)$. Then the set $S = \{x \in X / \omega_1^q(e)e^{2\pi i \alpha^q(e)} = \omega_1^q(x)e^{2\pi i \alpha^q(x)} \text{ and } \omega_2^q(e)e^{2\pi i \beta^q(e)} = \omega_2^q(x)e^{2\pi i \beta^q(x)}\}$ is a normal subgroup of X , where 'e' is the identity element of X .

Proof: At first $e \in X$, (i.e.) S is not empty. Moreover, it is subgroup of X , by theorem-3.9 so that

$$\omega_1^q(e)e^{2\pi i \alpha^q(e)} = \omega_1^q(x.y^{-1})e^{2\pi i \alpha^q(x.y^{-1})} \text{ and}$$

$$\omega_2^q(e)e^{2\pi i \beta^q(e)} = \omega_2^q(x.y^{-1})e^{2\pi i \beta^q(x.y^{-1})}.$$

But, 'A' is complex q-rung orthopair fuzzy normal subgroup of X . Then,

$$\begin{aligned}
\omega_1^q(x.a^{-1})e^{2\pi i \alpha^q(x.a^{-1})} &= \\
\omega_1^q(a^{-1}.x)e^{2\pi i \alpha^q(a^{-1}.x)} \text{ and} \\
\omega_2^q(x.a^{-1})e^{2\pi i \beta^q(x.a^{-1})} &= \\
\omega_2^q(a^{-1}.x)e^{2\pi i \beta^q(a^{-1}.x)}.
\end{aligned}$$

Hence, $(a^{-1}.x) \in S$ and 'S' is a normal subgroup of X .

5. Effect of Homomorphism on Complex q-rung orthopair fuzzy subgroup structures

In this section, we investigate the effect of homomorphism on complex q-rung orthopair fuzzy subgroup.

Definition-5.1: A function $f: X \rightarrow Y$ be a homomorphism from X to Y . Let 'A' be a complex q-rung orthopair fuzzy subgroup of X and 'B' be a complex q-rung orthopair fuzzy subgroup of Y . Let $x \in X$ and $y \in Y$, then we have

$f(A)(y) = \{(y, f(J_A)(y), f(K_A)(y))\}$ is the image of 'A' where;

$$f(J_A^q) = \begin{cases} \sup_{x \in f^{-1}(y)} J_A^q(x), & f(x) = y \\ 0, & \text{otherwise} \end{cases}$$

$$f(J_A^q) = \begin{cases} \sup_{x \in f^{-1}(y)} \omega_{IA}^q(x) e^{2\pi i \left(\sup_{x \in f^{-1}(y)} \alpha_A^q(x) \right)}, & f(x) = y \\ 0e^{2\pi i 0}, & \text{otherwise} \end{cases}$$

$$f(K_A^q) = \begin{cases} \inf_{x \in f^{-1}(y)} K_A^q(x), & f(x) = y \\ 0, & \text{otherwise} \end{cases}$$

$$f(K_A^q) = \begin{cases} \inf_{x \in f^{-1}(y)} \omega_{2A}^q(x) e^{2\pi i \left(\sup_{x \in f^{-1}(y)} \beta_A^q(x) \right)}, & f(x) = y \\ 1, & \text{otherwise} \end{cases}$$

and the pre image set of B is $f^{-1}(B)(x) = \{(x, f^{-1}(J_B)(x), f^{-1}(K_B)(x))\}$, where

$$\begin{aligned} f^{-1}(J_B^q)(x) &= (J_B)^q(f(x)) \\ &= \omega_{IB}^q(f(x)) e^{2\pi i \alpha_B^q(f(x))}, \\ f^{-1}(K_B^q)(x) &= (K_B)^q(f(x)) \\ &= \omega_{2B}^q(f(x)) e^{2\pi i \beta_B^q(f(x))}, \text{ for all } x \in X. \end{aligned}$$

Lemma-5.2: Let $f: X \rightarrow Y$ be a homomorphism from group X to group Y , and let 'A' be a complex q-rung orthopair fuzzy subgroup of X and 'B' be a complex q-rung orthopair fuzzy subgroup of Y . Then

1. $f(J_A^q)(y) = f(\omega_{IA}^q)(y) e^{2\pi i f(\alpha_A^q)(y)}, \forall y \in Y$.
2. $f(K_A^q)(y) = f(\omega_{2A}^q)(y) e^{2\pi i f(\beta_A^q)(y)}, \forall y \in Y$.
3. $f^{-1}(J_B^q)(x) = f^{-1}(\omega_{IB}^q)(x) e^{2\pi i f^{-1}(\alpha_B^q)(x)}, \forall x \in X$.
4. $f^{-1}(K_B^q)(x) = f^{-1}(\omega_{2B}^q)(x) e^{2\pi i f^{-1}(\beta_B^q)(x)}, \forall x \in X$.

Proof:

1.
$$\begin{aligned} f(J_A^q)(y) &= \sup_{x \in f^{-1}(y)} \{J_A^q(x); f(x) = y\} \\ &= \sup_{x \in f^{-1}(y)} \{\omega_{IA}^q(x) e^{2\pi i \alpha_A^q(x)}; f(x) = y\} \\ &= \sup_{x \in f^{-1}(y)} \{\omega_{IA}^q(x)\} e^{2\pi i \left(\sup_{x \in f^{-1}(y)} \alpha_A^q(x) \right)} \\ &= f(\omega_{IA}^q)(y) e^{2\pi i f(\alpha_A^q)(y)} \end{aligned}$$
2.
$$\begin{aligned} f(K_A^q)(y) &= \inf_{x \in f^{-1}(y)} \{K_A^q(x); f(x) = y\} \\ &= \inf_{x \in f^{-1}(y)} \{\omega_{2A}^q(x) e^{2\pi i \beta_A^q(x)}; f(x) = y\} \\ &= \inf_{x \in f^{-1}(y)} \{\omega_{2A}^q(x)\} e^{2\pi i \left(\sup_{x \in f^{-1}(y)} \beta_A^q(x) \right)} \\ &= f(\omega_{2A}^q)(y) e^{2\pi i f(\beta_A^q)(y)} \end{aligned}$$
3.
$$\begin{aligned} f^{-1}(J_B^q)(x) &= (J_B)^q(f(x)) \\ &= \omega_{IB}^q(f(x)) e^{2\pi i \alpha_B^q(f(x))} \\ &= f^{-1}(\omega_{IB}^q)(x) e^{2\pi i f^{-1}(\alpha_B^q)(x)} \end{aligned}$$
4.
$$\begin{aligned} f^{-1}(K_B^q)(x) &= (K_B)^q(f(x)) \\ &= \omega_{2B}^q(f(x)) e^{2\pi i \beta_B^q(f(x))} \\ &= f^{-1}(\omega_{2B}^q)(x) e^{2\pi i f^{-1}(\beta_B^q)(x)} \end{aligned}$$

Example-5.3: Let $(Z_5, +_5)$ and $(Z, +)$ be complex q-rung orthopair fuzzy group, where we define $(Z_5, +_5)$ as an example-4.3. The map $f: (Z, +) \rightarrow (Z_5, +_5)$ is complex q-rung orthopair fuzzy homomorphism.

Consider $A = \{1, 2, 7, 9, 13, 14\} \subseteq Z$. Then $f(A) = (x, f(J_A)(x), f(K_A)(x))$ where as

$$\begin{aligned} 1. f(J_A^q)(y) &= \sup_{x \in f^{-1}(y)} \{J_A^q(x); f(x) \\ &= y \pmod{5}\} \\ &= \sup \{J_A^q(1), J_A^q(2), J_A^q(7), J_A^q(9), J_A^q(13), J_A^q(14)\} \end{aligned}$$

$$\begin{aligned}
&= \sup \left\{ \begin{array}{l} \omega_{IA}^q(1)e^{2\pi i \alpha_A^q(1)}, \omega_{IA}^q(2)e^{2\pi i \alpha_A^q(2)} \\ , \omega_{IA}^q(7)e^{2\pi i \alpha_A^q(7)}, \\ \omega_{IA}^q(9)e^{2\pi i \alpha_A^q(9)}, \omega_{IA}^q(13)e^{2\pi i \alpha_A^q(13)}, \\ \omega_{IA}^q(14)e^{2\pi i \alpha_A^q(14)} \end{array} \right\} \\
&= \sup \left\{ \begin{array}{l} \omega_{IA}^q(1), \omega_{IA}^q(2), \dots, \\ \omega_{IA}^q(14) \end{array} \right\} e^{2\pi i \sup \{ \alpha_A^q(1), \alpha_A^q(2), \dots, \alpha_A^q(14) \}} \\
&= \sup \{0.49, 0.81, 0.04\} e^{2\pi i \sup \{0.36, 0.64, 0.25\}} \\
&= 0.81 e^{2\pi i 0.64} \\
2. f(K_A^q)(y) &= \inf_{x \in f^{-1}(y)} \{K_A^q(x); f(x) \\
&= y(\bmod 5)\} \\
&= \inf \{K_A^q(1), K_A^q(2), K_A^q(7), K_A^q(9), K_A^q(13), K_A^q(14)\} \\
&= \inf \left\{ \begin{array}{l} \omega_{2A}^q(1)e^{2\pi i \beta_A^q(1)}, \omega_{2A}^q(2)e^{2\pi i \beta_A^q(2)} \\ , \omega_{2A}^q(7)e^{2\pi i \beta_A^q(7)}, \\ \omega_{2A}^q(9)e^{2\pi i \beta_A^q(9)}, \omega_{2A}^q(13)e^{2\pi i \beta_A^q(13)}, \\ \omega_{2A}^q(14)e^{2\pi i \beta_A^q(14)} \end{array} \right\} \\
&= \inf \left\{ \begin{array}{l} \omega_{2A}^q(1), \omega_{2A}^q(2), \dots, \\ \omega_{2A}^q(14) \end{array} \right\} e^{2\pi i \inf \{ \beta_A^q(1), \beta_A^q(2), \dots, \beta_A^q(14) \}} \\
&= \inf \{0.36, 0.25, 0.09\} e^{2\pi i \inf \{0.25, 0.16, 0.49, 0.09\}} \\
&= 0.09 e^{2\pi i 0.09}
\end{aligned}$$

Theorem-5.4: Let $f: X \rightarrow Y$ be a epimorphism from $(X, \circ_1)$ to $(Y, \circ_2)$ and let 'A' be a complex q-rung orthopair fuzzy subgroup of X . Then $f(A)$ is complex q-rung orthopair fuzzy subgroup of Y .

Proof: Since 'A' is complex q-rung orthopair fuzzy subgroup, the set

$$T_1 = \{(\omega, \omega_{IA}(x), \omega_{2A}(x)) / x \in X, 0 \leq \omega_{IA}^q + \omega_{2A}^q \leq 1\} \text{ and}$$

$$T_2 = \{(x, \alpha_A(x), \beta_A(x)) / x \in X, 0 \leq \alpha_A^q + \beta_A^q \leq 1\}$$

are the amplitude and phase terms of the complex q-rung orthopair fuzzy subgroup. Then by Lemma 5.2 ; f is epimorphism, we have

Case-1:

$$(i)(a) (f(\omega_{IA}))^q(y_1 \circ_2 y_2) = f(\omega_{IA}^q)(y_1 \circ_2 y_2)$$

$$\begin{aligned}
&\geq \min\{f(\omega_{IA}^q)(y_1), f(\omega_{IA}^q)(y_2)\} \\
&= \min\{f(\omega_{IA}^q)(y_1), f(\omega_{IA}^q)(y_2)\} \\
(b) (f(\omega_{2A}))^q(y_1 \circ_2 y_2) &= f(\omega_{2A}^q)(y_1 \circ_2 y_2) \\
&\leq \max\{f(\omega_{2A}^q)(y_1), f(\omega_{2A}^q)(y_2)\} \\
&= \max\{f(\omega_{2A}^q)(y_1), f(\omega_{2A}^q)(y_2)\} \\
(c) (f(\alpha_A))^q(y_1 \circ_2 y_2) &= f(\alpha_A^q)(y_1 \circ_2 y_2) \\
&\geq \min\{f(\alpha_A^q)(y_1), f(\alpha_A^q)(y_2)\} \\
&= \min\{f(\alpha_A^q)(y_1), f(\alpha_A^q)(y_2)\} \\
(d) (f(\beta_A))^q(y_1 \circ_2 y_2) &= f(\beta_A^q)(y_1 \circ_2 y_2) \\
&\leq \max\{f(\beta_A^q)(y_1), f(\beta_A^q)(y_2)\} \\
&= \max\{f(\beta_A^q)(y_1), f(\beta_A^q)(y_2)\} \\
(ii)(a) (f(\omega_{IA}))^q(y^{-1}) &= f(\omega_{IA}^q)(y^{-1}) = \\
f(\omega_{IA}^q)(y) &= (f(\omega_{IA}))^q(y) \\
(b) (f(\omega_{2A}))^q(y^{-1}) &= f(\omega_{2A}^q)(y^{-1}) = \\
f(\omega_{2A}^q)(y) &= (f(\omega_{2A}))^q(y) \\
(c) (f(\alpha_A))^q(y^{-1}) &= f(\alpha_A^q)(y^{-1}) = \\
f(\alpha_A^q)(y) &= (f(\alpha_A))^q(y) \\
(d) (f(\beta_A))^q(y^{-1}) &= f(\beta_A^q)(y^{-1}) = \\
f(\beta_A^q)(y) &= (f(\beta_A))^q(y).
\end{aligned}$$

Case-2: Using case-1 and lemma-5.2, we have

$$\begin{aligned}
(1) (f(J_A))^q(y_1 \circ_2 y_2) &= f(J_A^q)(y_1 \circ_2 y_2) \\
&= f(\omega_{IA}^q)(y_1 \circ_2 y_2) e^{2\pi i f(\alpha_A^q)(y_1 \circ_2 y_2)} \\
&\geq \min \left\{ \begin{array}{l} f(\omega_{IA}^q)(y_1) e^{2\pi i f(\alpha_A^q)(y_1)}, \\ f(\omega_{IA}^q)(y_2) e^{2\pi i f(\alpha_A^q)(y_2)} \end{array} \right\} \\
&= \min\{f(J_A^q)(y_1), f(J_A^q)(y_2)\} \\
&= \min\{(f(J_A))^q(y_1), (f(J_A))^q(y_2)\} \\
(2) (f(K_A))^q(y_1 \circ_2 y_2) &= f(K_A^q)(y_1 \circ_2 y_2) \\
&= f(\omega_{2A}^q)(y_1 \circ_2 y_2) e^{2\pi i f(\beta_A^q)(y_1 \circ_2 y_2)} \\
&\leq \max \left\{ \begin{array}{l} f(\omega_{2A}^q)(y_1) e^{2\pi i f(\beta_A^q)(y_1)}, \\ f(\omega_{2A}^q)(y_2) e^{2\pi i f(\beta_A^q)(y_2)} \end{array} \right\}
\end{aligned}$$

$$\begin{aligned}
&= \max\{f(K_A^q)(y_1), f(K_A^q)(y_2)\} \\
&= \max\{(f(K_A))^q(y_1), (f(K_A))^q(y_2)\} \\
(3) \quad &(f(J_A))^q(y^{-1}) = f(J_A^q)(y^{-1}) \\
&= f(\omega_{IA}^q)(y^{-1})e^{2\pi i f(\alpha_A^q)(y^{-1})} \\
&= f(\omega_{IA}^q)(y)e^{2\pi i f(\alpha_A^q)(y)} \\
&= f(J_A^q)(y) \\
&= (f(J_A))^q(y) \\
(4) \quad &(f(K_A))^q(y^{-1}) = f(K_A^q)(y^{-1}) = \\
&f(\omega_{2A}^q)(y^{-1})e^{2\pi i f(\beta_A^q)(y^{-1})} \\
&= f(\omega_{2A}^q)(y)e^{2\pi i f(\beta_A^q)(y)} \\
&= f(K_A^q)(y) = (f(K_A))^q(y). \text{The proof is followed.}
\end{aligned}$$

Theorem-5.5: Let $f: X \rightarrow Y$ be a homomorphism from $(X, \circ_1)$ to $(Y, \circ_2)$ and let 'B' be a complex q-rung orthopair fuzzy normal subgroup of Y . Then $f^{-1}(B)$ is complex q-rung orthopair fuzzy normal subgroup of X .

Proof: Let 'B' be a complex q-rung orthopair fuzzy normal subgroup of Y , so that sets T_1 and T_2 are amplitude and phase terms in the complex q-rung orthopair fuzzy normal subgroup, as in case-1 of the above theorem-5.4 then, according to the definition-5.1 and lemma 5.2, we have

$$\begin{aligned}
f^{-1}(J_B^q)(x \circ_1 y) &= (J_B)^q(f(x \circ_1 y)) \\
&= \omega_{IB}^q(f(x \circ_1 y))e^{2\pi i \alpha_B^q(f(x \circ_1 y))} \\
&= \omega_{IB}^q(f(y \circ_1 x))e^{2\pi i \alpha_B^q(f(y \circ_1 x))} \\
&= f^{-1}(\omega_{IB}^q)(y \circ_1 x)e^{2\pi i f^{-1}(\alpha_B^q)(f(y \circ_1 x))} \\
&= (J_B)^q(f(y \circ_1 x)) = f^{-1}(J_B^q)(y \circ_1 x) \\
f^{-1}(J_B^q)(x \circ_1 y) &= f^{-1}(J_B^q)(y \circ_1 x).
\end{aligned}$$

Also, by same strategy we can show that

$f^{-1}(K_B^q)(x \circ_1 y) = f^{-1}(K_B^q)(y \circ_1 x)$. Hence by proposition-4.4 result is follows.

6. Conclusion:

We have generalized the notion of complex q-rung orthopair fuzzy subgroup and discussed various algebraic structures of it. This is happened by applying phase terms of complex numbers to the q-rung orthopair fuzzy structures and its conditions. Therefore, complex q-rung orthopair fuzzy subgroup is considered a generalization of complex intuitionistic fuzzy subgroup and complex Pythagorean fuzzy subgroup. Some results between the content concept and complex Pythagorean fuzzy subgroup were improved and discussed. Also, normality, coset and homomorphism under complex q-rung orthopair fuzzy subgroups were discussed and their properties were analyzed

7. Future work:

We may extent the presented works to some algebraic notions, such as factor groups, rings, fields and integral domains.

8. Author Declaration

* We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed. We further confirm that the order of authors listed in the manuscript has been approved by all of us.

***Conflict of Interest:** The authors declare no conflict of interest.

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