Development of Culture-Based Local Instructional Theory through Broad Learning Trajectory of Squares and Rectangulars in Elementary School Students

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ABSTRACT

The understanding of the concept of square and rectangle area in elementary school students is still dominated by the use of procedural formulas so that students have difficulty connecting measurements using square units with formal mathematical representations. Previous research on Project-Based Learning (PjBL) and culture-based learning generally focused on improving learning outcomes, while studies on the development of students' conceptual understanding to produce Local Instructional Theory (LIT) are still limited. This study aims to analyze the development of students' conceptual understanding through culture-based learning trajectories and develop Local Instructional Theory for square and rectangular area learning in elementary schools. The research uses a Design Research approach which includes the preliminary design stage, design experiment (pilot experiment and teaching experiment), and retrospective analysis. The research participants consisted of 101 grade V elementary school students in Medan City. Data were collected through observations, interviews, student worksheets, project artifacts, and learning documentation, then analyzed qualitatively using retrospective analysis. The results of the study show that students' conceptual understanding develops gradually through the identification of geometric shapes in miniature North Sumatra traditional houses, measurements using square units, spatial structuring, the formation of multiplicative reasoning, generalization of broad formulas, the application of concepts in projects, and the conclusion of broad concepts. The pilot experiment identified the learning obstacles that became the basis for the reconstruction of the Hypothetical Learning Trajectory (HLT) into Students' Learning Trajectory (LT) which was empirically validated and synthesized into the Final Local Instructional Theory. This research contributed to the development of Design Research by showing that culture-based PjBL facilitates the transformation of HLT into LT and produces adaptive learning design principles for mathematics learning.

Keywords: Design Research; Local Instructional Theory; Learning Trajectory; Project-Based Learning; Culture-Based Learning; Square and Rectangular Area.

Introduction

Understanding of broad concepts is one of the fundamental competencies in mathematics learning in elementary school because it is the foundation for the development of measurement skills, spatial reasoning, geometry, and mathematical problem solving. Broad measurement is an important part of the measurement domain that allows students to quantify two-dimensional space and is the basis for learning more complex mathematical concepts, such as surface area, volume, geometric coordinates, and calculus (Smith et al., 2016). In addition to having an important role in the development of geometric concepts, a broad understanding also supports spatial visualization, estimation, mathematical representation, and decision-making skills in various contexts of daily life (Palobo et al., 2025; Runnalls & Hong, 2020). Therefore, the learning of square and rectangular areas should not be oriented towards the mastery of formulas alone, but needs to help students build an understanding that area is the measure of an area obtained through the process of measuring using standard units of area. This conceptual understanding allows students to explain the meaning of the formula of area, connect the relationships between units of area, spatial structure, and multiplication operations, and transfer this knowledge into new problem-solving situations (Idrus et al., 2022; McNeal et al., 2024). Thus, broad learning in elementary school needs to be designed in stages by linking students' concrete experiences of two-dimensional objects to formal mathematical representations so that the concepts obtained are not only procedural, but also conceptually meaningful (Isaac, 2005; Smith et al., 2016).

Although the broad concept has been taught since elementary school, various studies show that it is still one of the most difficult geometry topics for students to understand (Idrus et al., 2022; Smith et al., 2016). This difficulty is shown through the tendency of students to memorize the area formula without understanding the meaning of area measurement as the process of measuring an area using repeated square units (Palobo et al., 2025; Tan Sisman & Aksu, 2016). In addition, students often equate broad concepts with circumference, experience misconceptions about the relationship between the two, and use incorrect units when solving measurement problems (Abadi & Amir, 2022; Widjaja & Vale, 2021). Other research also shows that most students still rely on the strategy of counting each square of units individually without recognizing the spatial structure of the arrangement of rows and columns that underlie the formation of multiplication operations in broad formulas (Clements et al., 2018; Permatasari & Amir, 2023). As a result, the use of long ×wide formulas is often done mechanically without understanding the underlying mathematical reasons or the relationship between unit squares, spatial structures, and multiplication operations. These conditions show that students' learning barriers are not only related to the mastery of procedures, but also to the process of constructing concepts that have not been optimally developed (Milinia & Amir, 2022).

Various studies have developed a variety of learning approaches to help students build a more meaningful understanding of broad concepts. The use of concrete manipulatives, unit squares, geoboards, grid paper, Lego, culture-based media, and dynamic geometry software has been proven to improve students' visualization, spatial exploration, and understanding of the relationship between geometric shapes and the concept of broad measurement (Clements et al., 2018; Cueva and Susada, 2024; Ponte et al., 2023; Zuliana et al., 2018) Other research develops learning trajectories through various assignments (variation tasks), Realistic Mathematics

Education (RME) approaches, and didactical design to facilitate students to build broad concepts gradually through contextual activities that are close to their experiences (Ahadiya et al., 2023; Purwati et al., 2023; Rostina et al., 2023). In addition, several studies also utilize Discovery Learning and GeoGebra media to improve learning activities, visualization, and student learning outcomes on square and rectangular area materials (Khuriyathusyfah & Muslim, 2024; Rostina et al., 2023). Although these innovations have shown a positive impact on improving learning outcomes and student understanding, most of the research is still oriented towards testing the effectiveness of learning models, media, or tools, while the process of developing students' conceptual understanding during learning has not been revealed in depth.

The dominance of research oriented towards learning effectiveness causes a limited understanding of how students build broad concepts gradually throughout the learning process. In fact, meaningful mathematics learning is not only shown by increasing learning outcomes, but also by the ability to explain the change in students' ways of thinking from informal understanding to formal mathematical understanding (Clements et al., 2018; Idrus et al., 2022). In the context of broad learning, this development includes the process when students begin to calculate unit squares, recognize spatial structures in the form of rows and columns, understand multiplication relationships as the constituent of unit squares, and finally be able to generalize these relationships into square and rectangular area formulas. Therefore, research on learning trajectory is important because it is able to describe the development of students' strategies, representations, and mathematical reasoning during the learning process, as well as providing an empirical basis for the development of learning designs that are in harmony with students' thinking development (Beautiful & Noble Beautiful, 2025).

Although studies on learning trajectory in mathematics learning continue to develop, research that produces culturally-context-based Local Instructional Theory (LIT) is still relatively limited. Most studies develop learning trajectories through specific cultural contexts, but focus more on the preparation of learning activities or validation of learning tools rather than producing local learning theories that explain the development of students' thinking empirically (Kurniawati & Amir, 2022; Rawani et al., 2023; Risdiyanti & Prahmana, 2021). Research on the development of LIT has also been carried out on various mathematical topics, such as rows and series, geometric transformations, statistics, and calculus, but there are still few who have developed LIT for the learning of square and rectangular area concepts in elementary school students through the context of local culture (Amsari et al., 2022; Kurniadi et al., 2026; Utari et al., 2025). As a result, there is still limited research that explains how cultural experiences can serve as a conceptual bridge that connects students' informal experiences with broad concepts formally through systematically designed, tested, and revised learning trajectories. These conditions demonstrate the need for the development of culture-based LIT that not only describes learning activities, but also explains the development of students' understanding based on empirical evidence during the learning process.

One of the cultural contexts that has the potential to support the learning of broad concepts is the traditional house of North Sumatra. Various ethnomathematical studies show that traditional houses in North Sumatra, such as Rumah Bolon, Rumah Karo Garista, Maimun Palace, and various other cultural buildings, contain various geometric concepts in the form of squares, rectangles, trapezoids, triangles, symmetry, proportions, and patterns that can be used as a source of mathematics learning (Nisa & Hasibuan, 2025; Rizqi et al., 2022; Safriyanti & Yafizham, 2023). In addition to having mathematical value, traditional houses also represent the cultural identity of the people of North Sumatra so as to provide a learning experience that is close to the lives of students (Ansya et al., 2024; Ansya et al., 2023). Through the activities of identifying the shapes of flat buildings in the traditional house, measuring the area of certain areas, and estimating the material needs in making miniature traditional houses, students can connect measurement activities with mathematical concepts in a more meaningful way. Thus, culture not only serves as an illustration of learning, but also becomes an authentic context that helps students build a conceptual understanding of the area through a learning experience that is contextual and relevant to their cultural environment (Fitrianto & Farisi, 2025; Hikmawati et al., 2021)

The integration of cultural contexts becomes more meaningful when developed through the Project-Based Learning (PjBL) approach. Through PjBL, students do not only learn broad concepts through practice questions, but are actively involved in designing, building, measuring, estimating, and reflecting on the results of the projects they are working on. In the process of making miniature traditional houses, for example, students need to identify the shapes of flat buildings, determine the size of each part, calculate the area of the materials needed, and evaluate the results of their work. These activities allow students to gradually build relationships between visual representations, concrete measurements, and mathematical symbols so that broad concepts develop through authentic and meaningful learning experiences. The use of projects rooted in local culture also provides students with opportunities to develop conceptual understanding while increasing engagement, collaboration, creativity, and appreciation of regional culture (Fitrianto & Farisi, 2025; Hikmawati et al., 2021; June et al., 2025).

To explain how the development of understanding occurs, this study uses a Design Research approach that is oriented towards the development of Local Instructional Theory (LIT). In this approach, the learning process begins with the preparation of a Hypothetical Learning Trajectory (HLT), then tested through pilot experiments and teaching experiments, and refined through retrospective analysis to produce validated local learning theories (Amsari et al., 2022; Rawani et al., 2023). This approach allows researchers to compare the predicted learning trajectory with the actual learning trajectory of students so that they can explain how thinking strategies develop, learning barriers arise, and the form of scaffolding required during the learning process (Kurniadi et al., 2026; Nopriyanti et al., 2026; Utari et al., 2025). Thus, the resulting LIT not only contains the sequence of learning activities, but also provides a theoretical contribution to the mechanism for developing broad conceptual understanding through cultural contexts and Project-Based Learning, so that it can be a reference in the development of contextual mathematics learning in elementary schools.

Based on these theoretical and empirical gaps, this study aims to develop a culture-based Local Instructional Theory (LIT) for square and rectangular area learning through Project-Based Learning in elementary school students. In contrast to previous research that generally focused on the effectiveness of media, learning models, or the use of cultural context as learning illustrations, this study positions North Sumatra's traditional house as an authentic context that builds the overall trajectory of students' thinking development. In this trajectory, students gradually develop from recognizing geometric shapes in traditional houses, understanding area through iterations of unit squares, organizing unit squares into row and column structures (spatial structuring), building multiplicative reasoning as the basis of broad formulas, to applying these concepts to complete the project of making miniature traditional houses. Thus, the main contribution of this research is not only to produce empirically validated learning trajectories, but also to produce a Local Instructional Theory that explains how cultural context and Project-Based Learning can facilitate the development of students' conceptual understanding from informal experiences to formal mathematical representations. This contribution is expected to enrich the study of Design Research, especially the development of culture-based mathematics learning theories in measurement materials in elementary schools.

Based on this description, this study aims to develop and validate culture-based Local Instructional Theory for square and rectangular learning through Project-Based Learning in elementary school students using the context of North Sumatran traditional houses. In particular, this study seeks to explain how the development of students' conceptual understanding during the design learning trajectory and how the results of the analysis of the learning trajectory are used to compile a local learning theory supported by empirical evidence. Therefore, this study is directed to answer the following two research questions.

1. How does students' conceptual understanding develop through culture-based learning trajectories on square and rectangular broad materials?
2. What are the characteristics of culture-based Local Instructional Theory that is produced to support square and rectangular wide learning in elementary schools?

Method

This study uses a Design Research approach with a validation study type that aims to develop and validate culture-based Local Instructional Theory (LIT) for square and rectangular learning through Project-Based Learning (PjBL) in elementary school students. This approach was chosen because the purpose of the research is not only to test the effectiveness of a learning design, but also to deeply understand how students build a conceptual understanding of the area, develop mathematical thinking strategies, and experience changes in thinking during the learning process. Through Design Research, this research produces a theory of local learning based on empirical evidence regarding the relationship between instructional design, learning activities, the development of students' thinking strategies, and the form of pedagogical support needed during the learning process.

The research refers to the validation study framework proposed by (Akker et al., 2006), which consists of three main phases, namely preliminary design, design experiment, and retrospective analysis. The design experiment stage is carried out through two cycles, namely pilot experiment and teaching experiment, so that learning design can be tested, revised, and refined gradually based on the development of student understanding. Through an iterative design, implementation, analysis, and revision process, Hypothetical Learning Trajectory (HLT) is developed into Local Instructional Theory (LIT) which explains how students build the concept of square and rectangular areas through culture-based learning trajectories.

This research was carried out on grade V elementary school students in Medan City, North Sumatra. The selection of participants was carried out purposively by considering the implementation of the learning design and the characteristics of students who had studied the prerequisites of the measurement concept. The research involved 101 students of class V, consisting of 40 students in the pilot experiment stage and 61 students in the teaching experiment stage. The division aims to enable the process of validation and refinement of the Hypothetical Learning Trajectory (HLT) gradually before being implemented on a wider class scale. Grade V students were chosen because the concept of square and rectangular area is an essential competency at this level and is the basis for learning more complex geometry concepts. All participants came from elementary schools that had comparable learning characteristics and followed the same curriculum.

The cultural context used in this study is the traditional house of North Sumatra. The context was chosen because it has various components of flat buildings, such as squares, rectangles, triangles, and trapezoids, which allow students to meaningfully relate cultural experiences with the concept of large measurement. Learning is developed through the Project-Based Learning (PjBL) approach, where students not only learn the concept of breadth procedurally, but also apply it in a project to make miniature traditional houses. Through these activities, students identify geometric shapes, determine the size of each part, calculate the area of the field, and estimate the needs of the materials used so that the concept of breadth develops through authentic and contextual learning experiences.

The research is carried out through three main phases of Design Research which are interrelated and take place iteratively. The first stage is preliminary design. At this stage, a literature review was carried out on broad learning, measurement, learning trajectory, Project-Based Learning, ethnomathematics, and Local Instructional Theory, accompanied by curriculum analysis, student characteristics, and identification of relevant cultural contexts. Based on the results of the analysis, a Hypothetical Learning Trajectory (HLT) was prepared which contains learning objectives, sequence of activities, suspected student thinking strategies, possible learning obstacles, and forms of scaffolding that are estimated to be needed during learning.

The second stage is a design experiment, which consists of a pilot experiment and a teaching experiment. The pilot experiment is conducted to evaluate the implementation of the initial HLT, identify students' thinking strategies, analyze responses to learning activities, and find learning obstacles that arise during implementation. The results of the analysis at this stage are used to revise the HLT before it is applied to a wider class scale. Furthermore, the teaching experiment is carried out using the revised HLT. In this stage, students participate in a series of culture-based PjBL activities designed to facilitate the development of broad conceptual understanding. Learning activities include the introduction of geometric shapes in traditional houses, measurements using unit squares, organizing unit squares into row and column structures (spatial structuring), the development of multiplicative reasoning as the basis for broad formulas, and the application of broad concepts in estimating material needs in the project of making miniature traditional houses. During implementation, attention is directed to the development of thinking strategies, changes in mathematical representations, and the development of students' conceptual understanding.

The last stage is retrospective analysis, which is a thorough analysis of all research data by comparing the hypothetical learning trajectory (HLT) with the actual learning trajectory. The results of the analysis are used to refine the HLT so as to produce a Local Instructional Theory that explains the mechanism of development of students' understanding of the concept of square and rectangular area.

To obtain a comprehensive picture of the development of students' conceptual understanding, the research used various complementary data sources. Classroom observations were conducted throughout the learning process to document student involvement, interaction patterns, problem-solving strategies, mathematical representation development, and forms of scaffolding provided by the teacher. The observation data was reinforced through field notes and photo documentation during the implementation of learning. Semi-structured interviews were conducted with several students who were purposively selected to obtain more in-depth information about the reasoning process, thinking strategies, reasons for using certain strategies, and obstacles experienced when building broad concepts.

In addition, the research utilizes various learning documents as data sources, including Student Worksheets (LKPD), student work results, artifacts of miniature traditional house projects, learning videos, documentation of the learning process, and various versions of HLT that have been revised during the research. The integration of these various data sources allows triangulation so that the development of students' learning trajectories can be analyzed comprehensively.

Data analysis follows the principle of retrospective analysis in Design Research with the aim of comparing the learning process predicted in the Hypothetical Learning Trajectory (HLT) with the actual learning process shown by students during the learning implementation. The analysis is carried out through three stages. The first stage is to organize all research data which includes observation results, interview transcripts, student work, project artifacts, field notes, learning documentation, and assessment results. Furthermore, the data is categorized based on the stages of learning activities and the objectives of concept development that have been formulated in the HLT.

The second stage is to compare thinking strategies, mathematical representations, the development of conceptual understanding, and learning obstacles that arise during learning with the alleged learning development that has been formulated in the HLT. This comparison is used to identify the suitability and differences between the hypothetical learning trajectory and the actual learning trajectory of the students. The third stage is to interpret the results of the analysis to identify learning moments, changes in thinking strategies, the development of spatial reasoning and multiplicative reasoning, as well as the form of scaffolding that plays a role in helping students build broad concepts. The results of the interpretation are used to refine the HLT and formulate a culture-based Local Instructional Theory that explains how students develop from contextual experiences in traditional houses to formal understandings of square and rectangular areas. The resulting LIT not only describes the sequence of learning activities, but also explains the mechanism of the development of students' conceptual understanding and provides theoretical contributions to the development of culture-based mathematics learning in elementary schools.3. Results

Results

Development of Initial Hypothetical Learning Paths

The development of the Hypothetical Learning Trajectory (HLT) begins at the preliminary design stage which aims to design a hypothetical learning trajectory to help students build a conceptual understanding of square and rectangular areas gradually through culture-based Project-Based Learning (PjBL). The preparation of HLT is based on three main foundations, namely conceptual analysis of square and rectangular area material, learning needs analysis, and literature review and elementary school curriculum. These three foundations are used to formulate learning objectives, predict the development of students' thinking, identify possible learning obstacles, and determine the form of didactic anticipation needed during the learning process.

Conceptual analysis shows that the understanding of square and rectangular area is built through several interrelated concepts, namely area as the measure of a two-dimensional surface, the use of unit squares as units of measurement, generalization of the area formula from unit square calculations, comparison of areas between fields, and the application of the concept of width in real situations. The relationship between these concepts is the basis for compiling the sequence of learning activities so that students are not immediately introduced to the formula, but first build a broad meaning through the measurement process using unit squares before generalizing to the long × wide formula.

In addition to conceptual analysis, the results of the needs analysis show that extensive learning in schools is still dominated by conventional approaches centered on teachers, the use of textbooks, and practice questions. This condition causes students to lack opportunities to actively build understanding of concepts through meaningful experiences. The results of the preliminary study also show that students' numeracy literacy skills, especially in the aspects of formula, interpret, and evaluate, are still not developed optimally, while the use of learning media is still limited to simple teaching aids. Based on these findings, HLT is designed by integrating Project-Based Learning, the cultural context of North Sumatra, and the use of concrete, visual, and digital media so that students can build understanding of concepts through exploratory and collaborative activities.

In this study, North Sumatran traditional houses were chosen as the main learning context because they provide various representations of flat buildings, such as squares and rectangles, which can be observed directly on floors, walls, windows, bases, and other architectural elements. The cultural context not only serves as an illustration of learning, but also serves as a conceptual mediator that connects students' informal experiences with formal concepts regarding area measurement. Through the project of making miniature traditional houses, students have the opportunity to apply broad concepts in authentic situations, such as identifying the shape of a flat building, measuring miniature parts, comparing the area between sections, and estimating the needs of the materials used.

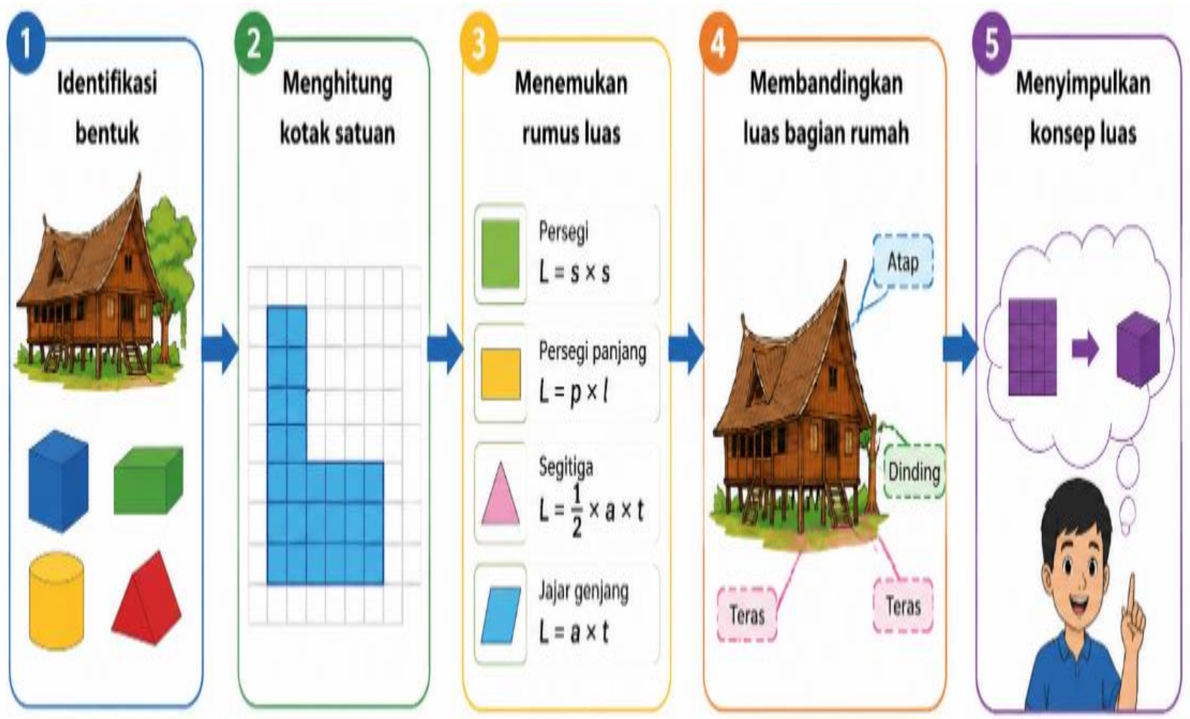


Figure 1. HLT Initial Square and Rectangle

Based on the results of the analysis, the initial HLT (Figure 1) was developed into five continuous learning activities. The first activity aimed to help students identify the shape of squares and rectangles, in miniature traditional houses. The second activity directed students to measure the area using unit squares through the process of calculating the number of squares that cover a field. The third activity facilitated students to find the relationship between the number of square units and the structure of rows and columns so that they could generalize the relationship into the formula of square and rectangular area. The fourth activity invited students to compare the area of different parts of the traditional house while estimating the material needs in the miniature making project. The last activity provided an opportunity for students to reflect on the learning outcomes and conclude that area is the number of square units that cover an area.

Each activity in HLT is designed based on four main components, namely learning objectives, learning activities, hypotheses of students' thought processes, and the teacher's didactic anticipation. For example, when students calculate area using unit squares, it is expected that some students will count boxes one by one, repeat the count on the same box, or skip multiple boxes. To anticipate these obstacles, the teacher provides scaffolding by having students mark the calculated boxes and direct the students' attention to the number of rows and columns through the guide questions. Similarly, when students begin to generalize the broad formula, the teacher facilitates the transition from the strategy of counting units to multiplicative reasoning through a discussion of the relationship between the number of rows, the number of columns, and the number of total square units.

The initial HLT predicted that students' understanding would develop gradually, starting from recognizing geometric shapes in traditional houses, measuring area using unit squares, organizing unit squares into spatial structures in the form of rows and columns (spatial structuring), building multiplicative reasoning as the basis for broad formulas, to applying broad concepts in the completion of the project of making miniature traditional houses. This hypothetical trajectory then became the basis for the implementation of pilot experiments and teaching experiments, and was refined through retrospective analysis to produce empirically validated Local Instructional Theory.

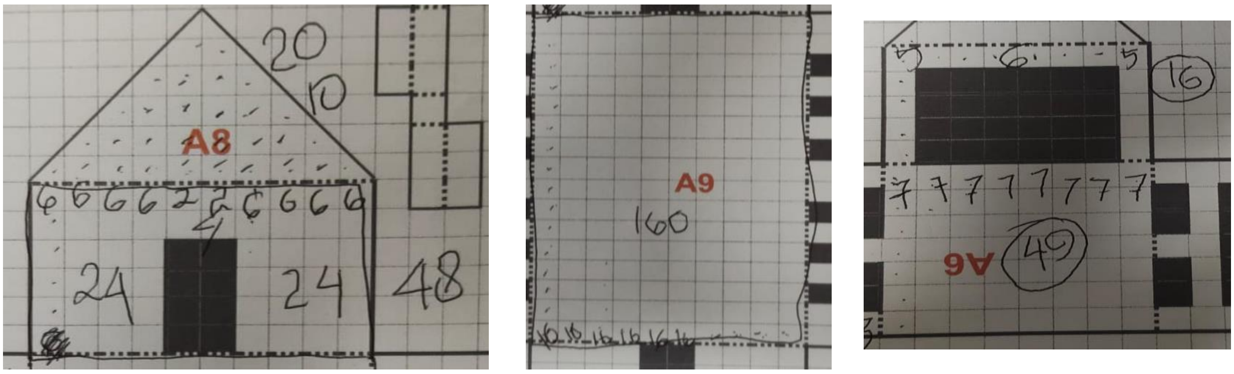
Introductory Experiment: Students' Early Learning Trajectory

The pilot experiment was carried out to evaluate the suitability of the initial Hypothetical Learning Trajectory (HLT) with the actual learning trajectory of students during the learning process. This stage aims to identify how students respond to each designed activity, uncover various learning obstacles that arise, and obtain an empirical basis for revising the HLT before it is implemented at the teaching experiment stage. The trial was carried out in small classes representing coastal, highland, and urban areas so that variations in students' socio-cultural characteristics can be observed during the learning process.

The results of the pilot experiment showed that the use of miniature North Sumatran traditional houses as a learning context was able to increase student involvement in all research areas. However, students' proximity to the cultural context did not directly result in mathematical understanding. Most students were able to recognize the square and rectangular parts of the traditional house, but still had difficulty transforming the shape into geometric representations used to determine the area. This condition shows that the process of mathematicizing from the cultural context to formal representation still requires the help of teachers through guiding questions and exploratory activities.

When students begin to determine the area using unit squares, a variety of different solving strategies emerge. Most students still use the counting by ones strategy, which is to count each unit square individually. However, an analysis of the student's work shows that the main obstacle lies not in the counting activity, but in the ability to organize the unit squares into an orderly spatial structure. As a result, students often count by jumping, recounting the same squares, or skipping several unit squares so that the calculation results become inconsistent. This obstacle shows that students have not been able to see the relationship between the number of rows, the number of columns, and the number of all unit squares as the basis for the formation of the concept of area.

Further analysis of the students' work results revealed three forms of obstacles that emerged during the pilot experiment. First, some students have been able to decompose the combined building into simple parts, but still have difficulty when determining the area of the triangle on the roof (Figure 2a). Students were able to calculate the area of the rectangle correctly, but were not able to identify the base and height of the triangle so they could not integrate the area of each part into the combined building area as a whole. These findings show that the process of mathematicization from visual representation to the formal concept of building area has not been optimally developed.



(a) (b) (c)

Figure 2. Students' answers calculate square area and length per cent

Second, some students directly use the length × width formula without going through the measurement process using unit squares (Figure 2b). Students write down the final result without showing the process of identifying the length, width, or number of unit squares that make up the building area. This indicates that the area formula is still understood as a memorized procedure, not as a generalization of the measurement process. Thus, the conceptual relationship between the activity of calculating unit squares and the use of the area formula has not been fully formed.

Third, there are students who have used the strategy of calculating unit squares by marking each row and column, but still make mistakes in determining the number of unit squares (Figure 2c). This error shows that students have not been able to organize unit squares into row and column structures systematically (spatial structuring). This obstacle causes some unit squares to be missed or counted more than once, so that the results of area measurements are inaccurate. The three findings show that students' mistakes are not solely caused by procedural weaknesses, but also by the lack of conceptual understanding of area as the size of an area.

These findings were further strengthened through interviews and observations during the lesson. When the teacher asked the students to find a more efficient way than counting one by one, most of the students were not able to relate the number of square units to the number of rows and columns. As a result, the transition process from the counting strategy to the structuring strategy did not take place as predicted in the initial HLT. The teacher then gave various forms of scaffolding, such as asking students to mark the squares that had been calculated, grouping the unit squares by rows and columns, and asking questions that related the calculation results to the multiplication operation. Through this process, some students began to realize that the broad formula is the result of generalizing the calculation of unit squares, not just rules that must be memorized.

As a result of the pilot experiment, there are three main learning obstacles that are the basis for the HLT revision, namely: (1) students have difficulty doing math from the cultural context to the representation of flat buildings, (2) students' spatial structuring skills have not developed so that the organization of unit squares into row and column structures is still not systematic, and (3) students have not been able to connect measurements using unit squares with the use of broad formulas as a form of mathematical generalization. Therefore, HLT is revised by adding a more gradual learning trajectory, starting from the identification of the areas to be measured, the organization of the unit squares, the development of relationships between rows and columns, to the generalization process towards the formula of square area, rectangles, and combined shapes. This revision further becomes the basis for the implementation of the teaching experiment as well as strengthening the development of Local Instructional Theory regarding broad learning based on the cultural context of North Sumatra.

Teaching Experiment: Improving Students' Learning Trajectory

Upland Student Learning Path

In the highland zone, the initial learning trajectory is designed for students to recognize the shape of squares and rectangles, to calculate the area using square units, to find the relationship between the unit square and the broad formula, to compare the area of several parts of the traditional house, and to conclude the concept of extent. However, the results of the pilot experiment show that the trajectory has not fully facilitated the development of students' conceptual understanding. Most students still view the activity of calculating unit squares and the use of long × wide formulas as two different procedures. The broad formula is understood more as a rule to be memorized than as a mathematical representation of the number of unit squares that cover a field. In addition, students also experience difficulties when asked to explain the reasons for using formulas or draw conclusions using their own language. The findings show that the initial learning trajectory still needs refinement, especially at the stage that connects concrete experiences with the process of generalizing broad concepts. Based on these learning obstacles, the learning trajectory is then reconstructed as the basis for implementing the teaching experiment.

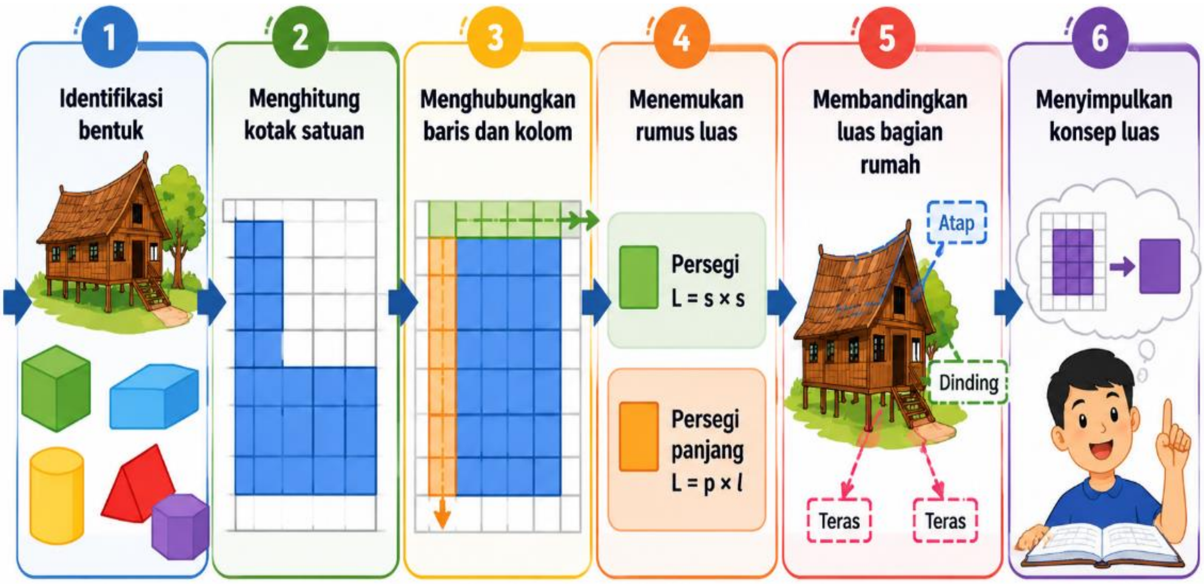


Figure 3. Learning Trajectory of Highland Students

Figure 3 shows the reconstructed learning trajectory developed based on the learning obstacles found in the pilot experiment. This reconstruction of the learning trajectory is designed to help students build a gradual relationship between concrete experiences and formal mathematical representations of the concept of vastness. The learning trajectory begins with the identification of shapes in miniature traditional houses as a learning context to recognize square and rectangular shapes. Next, students calculate the unit of boxes as a concrete representation of the area of a field. Based on this experience, students are directed to calculate repeatedly through the arrangement of rows and columns, so that they begin to recognize the regularity of the number of square units without having to count one by one. The next stage is to connect rows and columns to understand the relationship between the length, width, and number of unit squares that make up a field. Armed with this understanding, students then generalize the broad formula as a result of abstraction from the patterns they found themselves, apply it to compare the area of the various parts of the miniature traditional house, and finally deduce a broad concept based on the overall learning experience. Thus, the reconstructed learning trajectory serves as a conceptual bridge that connects concrete experiences to formal understanding before being implemented at the teaching experiment stage.

The implementation of the reconstructed learning trajectory at the teaching experiment stage showed a significant development in the way students think. The teacher no longer introduces the broad formula directly, but facilitates students to build understanding through the activity of organizing the unit squares into rows and columns. Through reflective questions and class discussions, students are directed to observe that the number of squares on each row and the number of rows form a relationship that can be represented through multiplication operations. At the same time, the teacher provides scaffolding in the form of an example of the arrangement of conclusions so that students are gradually able to explain the mathematical reasons for using formulas using their own language. As the learning progresses, students no longer just calculate the unit squares one by one, but begin to understand that the long × wide formulas are a form of generalization of the orderly arrangement of unit squares. This development is also followed by an increase in mathematical communication skills, which can be seen from the ability of students to explain the meaning of area as the number of square units that cover a surface along with the relationship between the structure of rows and columns with multiplication operations.

The results of the retrospective analysis showed that the implementation of the reconstructed learning trajectory resulted in a more comprehensive improvement in the student learning trajectory than the initial trajectory. If previously students moved directly from the activity of calculating square units to the use of broad formulas, then the learning trajectory resulting from the teaching experiment developed into more systematic stages, namely recognizing geometric shapes in the context of traditional houses, identifying areas to be measured, measuring using unit squares, organizing unit squares through row and column structures, building multiplication reasoning, generalizing broad formulas, comparing areas in cultural contexts, and communicating the meaning of broad concepts. The improvement of the learning trajectory shows that the addition of conceptual stages and mathematical communication plays an important role in helping students build a more meaningful understanding of broad concepts as well as being the basis for the preparation of Local Instructional Theory in the final stage of research.

Urban Student Learning Path

The development of students' learning trajectories in urban zones shows different characteristics compared to highland zones. The results of the pilot experiment show that most students are familiar with the formula of square and rectangular areas, but still maintain the strategy of calculating unit squares one by one when determining the area of a field. The findings show that students have been able to obtain measurement results, but have not yet understood that the arrangement of unit squares in rows and columns forms a structure that can be represented through multiplication operations. As a result, the area formula is still understood as a procedure used after the calculation process is completed, not as a representation of the pattern of the unit square arrangement. Based on these learning obstacles, the learning trajectory is then reconstructed by adding stages that strengthen the transition from counting activities to spatial reasoning and multiplicative reasoning as the basis for the teaching experiment.

Figure 4. Learning Trajectory of Urban Students

Figure 4 shows the learning trajectory of the reconstruction developed based on the learning obstacles found in the pilot experiment. This reconstruction of the learning trajectory is designed to help students build a gradual relationship between the concrete experience of calculating square units and a formal understanding of the concept of width. The learning trajectory begins with the identification of shapes on miniature traditional houses to recognize squares and rectangular parts. Next, students calculate square units as concrete representations of the area of a field. Based on this activity, students are directed to calculate repeatedly through the arrangement of rows and columns, so that they begin to recognize the pattern of regularity of the number of square units in each row. This experience is then continued with the activity of connecting rows and columns, so that students understand that the number of squares in each row and the number of rows form a relationship that can be represented through multiplication operations. Armed with this understanding, students then generalize the broad formula, apply it to compare the area of various parts of the miniature traditional house, and finally conclude a broad concept based on the overall learning experience. Thus, the reconstructed learning trajectory serves as a conceptual bridge that connects concrete experiences to formal understanding before being implemented at the teaching experiment stage.

The implementation of the reconstructed learning trajectory at the teaching experiment stage showed a real change in students' thinking strategies in determining the area of a field. At the beginning of learning, some students still used the counting by ones strategy, which is to count each square unit separately. However, after participating in an activity that emphasized the organization of unit squares through the arrangement of rows and columns, students began to switch to using a more efficient strategy by determining the number of squares in each row and multiplying it by the number of rows. The shift in strategy showed that students began to understand area as a mathematical structure built from the regularity of the arrangement of square units, not just the result of counting many objects. Through class discussions and reflective questions, students also began to be able to explain that multiplication operations are a representation of the relationship between the length, width, and number of square units that make up a field.

The results of the retrospective analysis showed that the implementation of the reconstructed learning trajectory resulted in an improvement in the student's learning trajectory compared to the initial trajectory. The improvement was marked by the emergence of spatial structuring stages that had not previously been developed in the pilot experiment. This stage functions as a conceptual bridge that connects the activity of calculating unit squares with the formation of multiplication reasoning before students generalize the broad formula. Thus, the learning trajectory of students in urban zones develops into more systematic stages, namely recognizing geometric shapes, calculating unit squares, organizing unit squares through row and column structures, building multiplication reasoning, generalizing broad formulas, applying broad concepts in cultural contexts, and deducing the meaning of broad concepts based on the learning experiences they have built.

Coastal Students' Learning Path

Based on the learning obstacles found in the pilot experiment, the learning trajectory of students in the coastal zone was reconstructed using the same stages as in the highland zone, namely identifying shapes, calculating unit squares, connecting rows and columns, finding broad formulas, comparing the area of the house, and deducing the concept of extent. The similarity shows that both zones require a learning trajectory that moves gradually from concrete experiences to formal mathematical representations. However, the didactic objectives of each stage in the coastal zone have a different emphasis than the highland zone.

In the highland zone, the reconstruction of the learning trajectory is focused on strengthening the conceptual relationship between the unit square and the broad formula and improving the students' ability to communicate the mathematical reasons for using the formula. In contrast, in the coastal zone, the reconstruction is more directed at building a systematic measurement process before students generalize the broad formula. Therefore, the teacher pays greater attention to the way students calculate the square of the unit in order, avoiding missed or double-counted boxes, and guiding students to relate the measurement results to the arrangement of rows and columns. This emphasis aims to make students understand that the long × wide operations are a representation of the number of square units arranged in an orderly manner, not just a calculation procedure.

Thus, although the structure of the learning trajectory in the coastal zone is the same as in the highland zone, the didactic function of each stage undergoes adjustment based on the characteristics of the learning obstacles found during the pilot experiment. The learning trajectory in the coastal zone emphasizes the formation of precision in the measurement process as a prerequisite for the development of spatial reasoning and multiplicative reasoning. This approach allows students to build a conceptual understanding of the area through systematic measurement experiences before generalizing the broad formula and applying it in the context of a miniature traditional house.

Development of the Final Local Instructional Theory

The results of the pilot experiment, teaching experiment, and retrospective analysis show that the understanding of the concept of square and rectangular area develops through a construction process that takes place gradually from concrete experience to formal mathematical representation. Although the implementation was carried out on three different regional characteristics, namely highlands, urban, and coastal, all three showed similar conceptual development patterns. The differences that emerged did not lie in the order of learning activities, but in the characteristics of the learning obstacles faced by students as well as the didactic emphasis given by teachers during the learning process. These findings show that one learning trajectory can be applied to different contexts as long as each stage is adjusted to the conceptual needs of the students.

Cross-zone analysis showed that the improvement of the learning trajectory did not require a change in the order of activities, but required adjustments to the didactic function of each stage. In the highland zone, learning was focused on strengthening the conceptual relationship between the unit square and the broad formula as well as the development of mathematical communication so that students were able to explain the reasons for using formulas. In urban zones, emphasis was placed on the development of spatial structuring, which is the ability to organize unit squares into rows and columns so that students move from the strategy of counting by ones to multiplicative reasoning. Meanwhile, in the coastal zone, emphasis is directed to the formation of a systematic measurement process through sequential calculation of unit squares so that students understand that long ×wide operations are a representation of the number of square units arranged in a field. The difference in emphasis shows that the learning trajectory is adaptive to the characteristics of students' learning barriers, although the structure of the learning trajectory used remains the same.

Based on the synthesis of the three implementations, this research produced a Final Local Instructional Theory which explains that the learning of square and rectangular areas based on cultural context develops optimally if students are guided through a learning trajectory that moves from concrete activities to mathematical abstraction gradually. The trajectory begins with the identification of shapes in miniature traditional houses as a cultural context to build initial mathematicalization. Next, students calculate unit squares as concrete experiences in understanding area as the measure of a surface. After gaining this experience, students are directed to connect the arrangement of square units in rows and columns, so that they begin to recognize the regularity of the spatial structure that underlies the multiplication operation. Based on these relationships, students generalize the formula of area as a mathematical representation of the number of square units, then apply the concept to compare the area of various parts of the miniature traditional house, and finally conclude the concept of area based on the results of exploration and discussion that has been carried out. This trajectory shows that broad formulas are not provided as initial information, but are built through a process of guided reinvention based on the measurement experiences students experience.

The main characteristic of the resulting local learning theory lies not only in the use of miniature traditional houses as a learning context, but also in the integration between cultural experience, concrete measurement, spatial structuring, multiplicative reasoning, and mathematical communication in one complete learning trajectory. The cultural context serves as the starting point for mathematicization, the square of units becomes the basis for the formation of broad meanings, the row and column structure connects concrete experiences with multiplication operations, while activities of comparing breadth and concluding concepts strengthen students' ability to reflect on the relationship between concrete representations and formal representations. Such integration allows students to build a more meaningful conceptual understanding than learning that directly introduces broad formulas as a procedure.

Thus, the Final Local Instructional Theory produced by this study is not only in the form of a sequence of learning activities, but is a didactic principle that explains how broad concepts are constructed through the interaction between cultural contexts, measurement experiences, and the gradual process of mathematicalization. This theory emphasizes that the success of broad learning does not depend on providing formulas directly, but on the teacher's ability to design scaffolding that suits students' learning obstacles so that each learning stage becomes a bridge to the next stage. Therefore, the resulting local learning theory has the potential to be adapted to geometry learning in other cultural contexts by maintaining the structure of the learning trajectory, but adjusting the didactic emphasis according to the characteristics of the learning barriers faced by students.

Discussion

This research shows that culture-based Project-Based Learning (PjBL) not only functions as a learning model, but also as a learning environment that allows students to build a broad understanding of concepts through authentic and meaningful learning experiences. In contrast to procedural-oriented learning, PjBL places students as the main actors who actively explore, investigate, collaborate, and produce solutions to real problems through a project (Rehman et al., 2024; Sánchez-García & Reyes-de-Cózar, 2025). In this study, the project of making and analyzing miniature traditional houses in North Sumatra provides students with the opportunity to identify geometric shapes, measure the area of various parts of the building, compare the measurement results, and discuss the solution strategies used. This series of activities makes the concept of a broad concept not studied as a set of formulas that must be memorized, but as a mathematical tool to solve problems that arise during the project process. These findings are in line with Chang et al., (2024) which affirms that authentic context in PjBL is able to increase student engagement, strengthen interaction during learning, and help students build deeper conceptual understanding. Similarly, Ngereja et al., (2020) Shows that project-based learning experiences have a positive impact on motivation, learning quality, and competency development relevant to real life.

The findings of the study also show that the cultural context does not only function as a learning medium, but as an authentic context that mediates the process of constructing mathematical knowledge. Miniature North Sumatran traditional houses provide real objects that allow students to connect everyday experiences with formal concepts regarding square and rectangular areas. Through the activities of observing the shape of the building, determining the parts to be measured, calculating the area of walls, roofs, and terraces, students gain concrete experiences that become the basis for the process of abstraction of broad concepts. The results of this study support the study Luzano (2025) which states that culturally responsive mathematics learning is able to increase engagement, understanding, and inclusivity because it links mathematical concepts to students' cultural experiences. These findings are also in line with Dhema et al., (2025) which confirms that the integration of ethnomathematics and local wisdom creates more meaningful learning while strengthening students' cultural identities. However, the results of the pilot experiment in this study show that the existence of cultural context alone does not automatically result in mathematical understanding. Some students still have difficulty connecting the activity of counting unit squares with broad formulas or organizing unit squares into row and column structures. Therefore, the effectiveness of culture-based learning is not only determined by the selection of authentic contexts, but also by how teachers design activities that are able to bridge concrete experiences to formal mathematical representations.

Furthermore, this study shows that the success of culture-based PjBL is not only determined by the characteristics of the project or the cultural context used, but also by the design of the learning trajectory that directs the development of students' thinking during the project process. Project activities provide concrete experiences as a starting point for learning, while the learning trajectory organizes the experience into systematic conceptual stages, starting from recognizing geometric shapes, measuring using square units, connecting the arrangement of rows and columns, generalizing broad formulas, applying concepts in the project, to concluding broad meanings. These findings show that the project functions as a learning context, while the learning trajectory functions as a pedagogical mechanism that ensures that each learning experience contributes to the conceptual development of students. This interpretation expands the results of the research Gannar & Kilani (2025), which shows that a project-based approach with contextual activities and repetitive reflection encourages the construction of knowledge gradually, as well as being in line with the PjBL implementation framework put forward by Sánchez-García & Reyes-de-Cózar (2025), which emphasizes the importance of project design, reflection, and continuous evaluation in producing meaningful learning. Thus, culture-based PjBL in this study not only produces more contextual learning, but also becomes a vehicle to reconstruct the Hypothetical Learning Trajectory (HLT) into an empirically validated Students' Learning Trajectory (LT) and subsequently develop into a Final Local Instructional Theory (LIT).

One of the main findings of this study is that the Hypothetical Learning Trajectory (HLT) designed at the preliminary design stage has not fully represented the development of students' thinking during the initial implementation. In the framework of Design Research, HLT is indeed positioned as a hypothesis regarding how the learning process is expected to develop, so its validity must be proven and refined through direct interaction with students during the design experiment (Baroody et al., 2022; Duschl et al., 2011; Fauziyah & Husniati, 2023). The results of the pilot experiment in this study show that students still face various learning obstacles that hinder the transition from concrete experience to formal understanding. These obstacles include the tendency to count by ones, not being able to organize the unit squares into row and column structures, and viewing broad formulas as procedures that must be memorized without understanding their conceptual relationship with the measurement process. These findings are in line with Smith et al., (2016) which explains that one of the main difficulties students in studying area is understanding how the multiplication of length and width represents the structure of two-dimensional space, as well as by Asil Güzel & Yeşildere İmre (2024) which shows that students often rely on formulas without having adequate reasoning about the meaning of broad measurements. In addition, the results of this study also corroborate the findings Stuart & Stuart (2025) that epistemological obstacles are still the main cause of students' low reasoning ability in square and rectangular material.

These findings became the basis for the reconstruction of HLT through the addition of learning stages that explicitly strengthened the relationship between unit squares, spatial structuring, multiplicative reasoning, and mathematical communication. This revision not only changed the order of activities, but also improved the anticipation of student responses and teachers' didactic strategies so that each activity functions as a bridge to the next stage of thinking. The results of the teaching experiment showed that the improvement resulted in a Students' Learning Trajectory (LT) that was more in line with the students' conceptual development. Students began to move from the strategy of counting by ones to organizing unit squares based on row and column structures, then used the structure to construct multiplication reasoning and generalize the broad formula independently. This change shows that the broad formula is no longer understood as a stand-alone procedure, but as a mathematical representation of the arrangement of the unit squares that they have constructed previously. These results support the view Baroody et al., (2022) that the effectiveness of HLT lies in its ability to connect learning objectives, student thinking development, and learning activities in an integrated manner. These findings are also in line with Fauzan & Diana (2020), Marande & Diana (2023), Fadilah et al., (2024), and Patmala et al., (2025) which shows that the revision of HLT based on the results of the pilot experiment results in a more realistic learning trajectory and is more in line with the way students think than the initial design.

Another important finding is that the reconstructed structure of the learning trajectory can be applied consistently to three different regional characteristics, namely highlands, urban, and coastal. The difference in implementation does not lie in the order of learning activities, but in the didactic emphasis given by the teacher in response to the characteristics of learning obstacles in each zone. In the highland zone, teachers emphasize more on strengthening the conceptual relationship between the square of units and the broad formula as well as mathematical communication skills. In the urban zone, learning is focused on developing spatial structuring so that students are able to move from the counting by ones strategy to multiplicative reasoning. Meanwhile, in the coastal zone, teachers pay more attention to the establishment of a systematic measurement process so that students do not miss or recalculate the unit square before generalizing the broad formula. These findings show that the flexibility of the learning trajectory is not always realized through changes in the structure of activities, but through the adaptation of didactic strategies based on students' learning needs. This interpretation supports the concept of adaptive teaching which emphasizes the importance of teachers' ability to adjust learning decisions based on student responses and needs during the learning process (Gallagher et al., 2022). Thus, this study shows that the transformation of HLT into Students' Learning Trajectory is an iterative process that not only perfects the learning design, but also produces didactic principles that are the basis for the development of the Final Local Instructional Theory (LIT).

The main contribution of this research is the development of Final Local Instructional Theory (LIT) which explains how the understanding of the concept of square and rectangular areas can be constructed through the integration of Project-Based Learning (PjBL), the cultural context of North Sumatra traditional houses, and learning trajectories that have been empirically validated through the Design Research approach. In contrast to the Hypothetical Learning Trajectory (HLT) which is still a theoretical conjecture about the possibility of student thinking development, the Final Local Instructional Theory produced is a synthesis of the entire research process, starting from preliminary design, pilot experiment, HLT reconstruction based on learning obstacles, teaching experiment, to retrospective analysis. Thus, the resulting theory not only describes the sequence of learning activities, but also explains the relationship between learning objectives, student thinking development, student response anticipation, and teacher didactic decisions that allow the gradual construction of concepts. These findings reinforce the view that Local Instructional Theory is the main product of Design Research that was born through an iterative and reflective design process, not just the result of the implementation of a learning tool (Amsari et al., 2022; Baroody et al., 2022; Doorman, 2019; Nopriyanti et al., 2026; Nur Atiqoh & Hafiz, 2025).

The resulting Local Instructional Theory shows that the learning of broad concepts develops through a progressive trajectory from the identification of geometric shapes in miniature traditional houses, measurements using unit squares, organization of unit squares through spatial structuring in the form of rows and columns, the formation of multiplicative reasoning, generalization of broad formulas, the application of concepts in the project of comparing the area of parts of traditional houses, to the formal conclusion of broad concepts. This trajectory shows that the broad formula was not introduced as a procedure that must be memorized from the beginning, but emerged as an abstraction from the measurement experience carried out by the students themselves through culture-based project activities. In other words, the concrete experience gained through the exploration of traditional houses serves as a bridge to formal mathematical representation so that the mathematicization process takes place gradually. These findings support the results of the research Clements et al. (2018), Cullen et al., (2018) and McNeal et al. (2024) which confirms that the understanding of broad concepts develops when students are able to organize the square of units into spatial structures and relate the structure to multiplication reasoning before using formulas formally. In addition, the learning trajectory developed also strengthens the results of the research Kurniawati & Amir (2022) and Purwati et al., (2023) which shows that broad learning becomes more meaningful when concepts are built through learning trajectories that develop from concrete activities to symbolic representations.

Another theoretical contribution lies in the finding that the transformation of HLT into a Students' Learning Trajectory does not require a change in the basic structure of the learning trajectory, but rather a change in the didactic emphasis according to the characteristics of students' learning obstacles. In all three study zones—highlands, urban, and coastal—the order of learning activities was maintained, but teachers provided different forms of scaffolding, representations, triggering questions, and conceptual support according to the needs of students in each region. Thus, learning flexibility is not realized through changes in activities, but through the ability of teachers to adapt learning strategies based on students' actual responses. These findings expand the study of adaptive teaching in Design Research by showing that didactic adaptation is the main mechanism that bridges HLT to students' actual learning trajectories without eliminating learning design coherence (Gallagher et al., 2022)..

In addition to contributing to the development of local learning theories, this study also expands the use of ethnomathematics-based Project-Based Learning as a vehicle to construct mathematical concepts. Previous studies have generally reported that cultural integration in PjBL contributes to increasing creativity, critical thinking, mathematical disposition, and conceptual understanding of students (Kamil et al., 2025; Ngala & Marsigit, 2024; Pradhan & Basnet, 2025; Syarifuddin et al., 2025; Wikarsari et al., 2025; Zhao, 2024). However, the research still focuses on learning outcomes, while the mechanism of student thinking development during the learning process has not been widely explained. This research complements this gap by showing how the cultural context of traditional houses not only functions as a contextual medium, but also becomes a didactic instrument that connects students' concrete experiences with the development of learning trajectories towards formal understanding. Therefore, the resulting Final Local Instructional Theory is not only relevant for extensive learning based on North Sumatran traditional houses, but also offers a set of learning design principles that can be adapted to various other cultural contexts by maintaining the structure of the learning trajectory and adjusting didactic strategies based on the characteristics of students' learning obstacles.

Conclusion

This research answers research questions about how students' conceptual understanding develops through culture-based learning trajectories on square and rectangular wide materials and how the characteristics of Local Instructional Theory (LIT) are produced to support this learning. The results of the study show that students' conceptual understanding develops through a learning trajectory that moves gradually from concrete experiences to formal mathematical representations. The trajectory begins with the identification of shapes in miniature North Sumatra traditional houses, followed by measurements using square units, organization of unit squares through row and column structures (spatial structuring), the formation of multiplicative reasoning, generalization of broad formulas, the application of concepts in the project, and the conclusion of broad concepts. This development occurs through the process of reconstructing the Hypothetical Learning Trajectory (HLT) based on learning obstacles found during the pilot experiment and re-validated through the teaching experiment, resulting in a Students' Learning Trajectory (LT) that is more in line with the development of students' thinking.

The main theoretical contribution of this research is the development of Final Local Instructional Theory (LIT) which explains the mechanism for developing broad conceptual understanding through the integration of Project-Based Learning (PjBL), cultural contexts, and empirically validated learning trajectories. In contrast to previous research which generally focused on the effectiveness of PjBL or the use of culture as a learning context, this study shows that students' conceptual development takes place through the process of transforming HLT into Students' Learning Trajectory, which is then synthesized into Final Local Instructional Theory through retrospective analysis. This study also shows that the structure of learning trajectories can be maintained on various regional characteristics, while adaptation is carried out through adjustment of didactic emphasis according to the characteristics of students' learning obstacles. The findings expand the study of Design Research by emphasizing that the flexibility of learning trajectories does not depend on changes in the order of learning activities, but on the ability of teachers to reconstruct didactic decisions based on students' learning responses.

Practically, the results of this study imply that culture-based Project-Based Learning can be used as a learning environment that supports the development of students' conceptual understanding through authentic and meaningful learning experiences. The resulting Final Local Instructional Theory provides learning design principles that can be used as a reference by elementary school teachers in designing square and rectangular learning that is centered on the concept construction process, not just the use of procedural formulas. In addition, this local learning theory has the potential to be adapted to geometry and other mathematics topics by utilizing different local cultural contexts, as long as the structure of the learning trajectory is maintained and the didactic emphasis is adjusted to the characteristics of students' learning obstacles. Further research can expand the validation of this Local Instructional Theory at the educational level, cultural context, and other mathematical concepts

to test transferability and enrich the development of Design Research-based learning theory.

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