Accessible Teaching of Quadratic and Hyperbolic Functions for Learners with Disabilities and Mathematics Learning Difficulties: An Integrative Review and Inclusive Instructional Framework

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ABSTRACT:

Quadratic and rectangular-hyperbolic functions are central to secondary mathematics, yet their combined symbolic, graphical, linguistic and multi-step demands can create disproportionate barriers for learners with disabilities and persistent mathematics learning difficulties. This critical integrative review synthesises function-specific studies with evidence from special and inclusive mathematics education. Literature published mainly from 2015 to July 2026 was identified through ERIC, DOAJ, PubMed Central, publisher platforms, official policy sources and backward reference checking. The synthesis shows that difficulty emerges from interactions among fragile prerequisite knowledge, working-memory load, representational discontinuity, mathematical language, anxiety and inaccessible assessment. Promising responses combine explicit and systematic instruction with conceptual explanation, cognitive offloading, connected representations, dynamic graphing, diagnostic error analysis, structured peer explanation and flexible assessment. Evidence specifically involving learners with identified disabilities remains limited for these advanced function topics; consequently, causal claims must remain cautious. The article proposes the ACCESS-QH framework: Assess, Chunk and cue, Connect representations, Enable access, Scaffold strategy and language, and Support self-regulation. The framework translates special-education principles into rigorous classroom planning without reducing the intended mathematical demand.

Keywords: Quadratic functions, hyperbolic functions, learning disabilities, inclusive mathematics education, Universal Design for Learning.

1. Introduction

Access to advanced school mathematics is an inclusion issue as well as a curriculum issue. Learners who experience mathematics learning disabilities or persistent mathematics difficulties are frequently expected to engage with the same abstract content as their peers while simultaneously managing slower processing, limited working memory capacity, difficulty retrieving prerequisite knowledge, language barriers, executive function demands, and mathematics anxiety. These barriers do not imply an inability to reason mathematically. They indicate that the conventional instructional design may create unnecessary obstacles between the learner and the intended mathematical idea. Quadratic and hyperbolic functions are especially revealing cases. Quadratic functions require learners to coordinate equations, tables, graphs, roots, turning points, axes of symmetry, factorisation, completing the square and contextual modelling. Hyperbolic functions in the South African secondary-school sense require learners to reason about reciprocal relationships, excluded values, horizontal and vertical asymptotes, translations and the effects of parameters on two disconnected branches. In both topics, success depends on moving flexibly between symbolic, graphical, numerical and verbal representations. A learner may execute one procedure yet remain unable to explain what a root, a vertex, or an asymptote means. In South African school mathematics, the term hyperbolic function normally refers to the translated rectangular-hyperbola family $f(x) = a/(x + p) + q$, rather than the tertiary hyperbolic trigonometric functions sinh and cosh. The present article uses this curriculum's meaning. The distinction matters because the instructional demands concern reciprocal graphs, excluded values, asymptotes, translations and two disconnected branches. The International Journal of Special Education provides an appropriate forum for this inquiry because the problem extends beyond mathematics attainment. It concerns whether learners with diverse support needs receive meaningful access to a full secondary curriculum, including abstract function content. The review, therefore, treats disability as an interaction between learner characteristics and the accessibility of teaching, representation, technology, participation and assessment. The review was guided by three objectives: (1) to identify cognitive, representational, affective and contextual barriers shaping the learning of quadratic and rectangular-hyperbolic functions; (2) to evaluate instructional approaches supported by function-specific research and the wider special-education evidence base; and (3) to organise the evidence into a classroom-facing inclusive instructional framework. The central question was: How can quadratic and rectangular-hyperbolic functions be taught accessibly without reducing their mathematical rigour?

1.1 Literature review: Quadratic and hyperbolic functions, learner diversity and inclusion

Quadratic work often combines several operations into a single task. Factorising ax^2+bx+c, solving an equation, interpreting roots as x-intercepts and connecting the result to a graph require learners to retain intermediate results while selecting and monitoring procedures. Hord, Christman, and Berman (2025) describe quadratic problems as vulnerable to cognitive overload and anxiety because of their multiple steps, abstract relationships, and dependence on foundational skills such as integer operations, distributions, and factor pairs. A South African study of 42 Grade 11 learners found that many could not describe the roots conceptually, reverse from roots to an equation, or determine an unknown parameter and a second root when one root was supplied (Makgakga, 2023). Hyperbolic functions add an unusual visual and conceptual structure. Matindike and Makonye (2023) analysed diagnostic tests and task-based interviews with 13 Grade 11 learners at a rural South African school. Learners displayed algebraic, conceptual, asymptotic, graphical and notational misconceptions, and their conceptions were largely limited to action-level procedures. The findings are not disability-specific, but the identified demands—fractional notation, excluded values, parameter shifts, asymptotic behaviour, and graph interpretation—are likely to be intensified for learners with working memory, language, or visual-spatial processing difficulties. An inclusive account must therefore resist two simplifications. First, it should not attribute every error to an internal deficit. Fast pacing, visually crowded worksheets, inaccessible graphing interfaces, unexplained vocabulary and assessment formats can create disability through design. Second, it should not assume that a general UDL label is sufficient. Learners may still require explicit, individualised and intensive instruction, accommodations or assistive technologies in addition to universally designed lessons. Secondary algebra research involving learners with learning disabilities is smaller than the literature on early numeracy, but available reviews identify a recurring cluster of effective features: explicit modelling, systematic sequencing, visual representations, opportunities for guided and independent practice, strategy instruction, corrective feedback and cumulative review. Bone, Bouck and Witmer (2021) reviewed 18 algebra-intervention studies and judged 14 to meet high-quality standards. Lee et al. (2020) similarly reported positive evidence for structured interventions targeting algebraic concepts and skills, while cautioning that the number and diversity of studies remain limited. These findings do not support a return to procedure without meaning. Learners need clear modelling of what to do and opportunities to understand why a method works. In quadratic functions, for example, the zero-product principle should be connected to graph intercepts; factor pairs should be connected to expansion; completing the square should be linked to vertex form; and parameter changes should be traced across equation, table and graph. Special education is strongest when accessibility and conceptual integrity are treated as mutually reinforcing rather than competing aims. Quadratic and rectangular-hyperbolic functions share the broader function concept of correspondence, covariation and parameter effects, but they place different demands on learners. A parabola is continuous across its domain and organised around a turning point and an axis of symmetry. A rectangular hyperbola contains an excluded input, vertical and horizontal asymptotes, and two separate branches. Teaching the topics comparatively can support classification only when their similarities and differences are made explicit. The literature also cautions against locating every error within the learner. Fast pacing, visually crowded worksheets, unexplained terminology, colour-dependent graphs, inaccessible software and assessment formats can create avoidable barriers. Conversely, a general claim that a lesson follows Universal Design for Learning does not remove the need for individualised, intensive instruction, accommodations and assistive technology where these are required.

1.2 Theoretical and conceptual framework

UDL proposes proactive design through multiple means of engagement, representation, and action and expression. In mathematics, this can include tactile or enlarged graphs, verbal descriptions of visual features, colour-independent line styles, manipulable digital graphs, worked examples, captioned demonstrations, choice in response format and explicit supports for planning and self-monitoring. UDL does not mean presenting the same content in many decorative forms; representations must reveal mathematically relevant structure. A meta-analysis by Almeqdad et al. (2023) synthesised 13 empirical studies published from 2015 to 2021 and found generally positive effects of UDL-oriented interventions, although the evidence base was heterogeneous. Root et al. (2020) demonstrated a functional relation between a UDL-aligned mathematics intervention and improved problem solving for three middle-school learners with extensive support needs. In South African rural mathematics classrooms, Moleko and Maphalala (2025) found that teachers associated UDL with flexible access, improved processing and broader participation. McKenzie and Dalton (2020) argued that UDL can provide a practical bridge between inclusive-education policy and curriculum differentiation in South Africa.

Cognitive Load Theory distinguishes the complexity inherent in a task from the load created by avoidable instructional design. Quadratic and hyperbolic tasks are intrinsically complex because several elements interact. Teachers cannot remove mathematics, but they can reduce extraneous load by segmenting procedures, keeping related representations spatially close, using consistent notation, fading prompts gradually and avoiding crowded pages or simultaneous verbal and symbolic overload (Sweller et al., 2019). Cognitive offloading is particularly relevant. Learners can be taught to externalise intermediate steps in organised spaces rather than hold them mentally. Hord et al. (2025) recommend structured scratchwork, factor-pair lists, box or window arrangements, and explicit checking routines. Such supports should be viewed as tools for reasoning, not signs of low ability. Once a learner becomes fluent, the supports can be faded or retained as legitimate accommodations. APOS theory describes conceptual development through actions, processes, objects and schemas. It is useful for distinguishing a learner who can substitute values only when prompted from one who can mentally coordinate how a parameter transforms an entire graph. Matindike and Makonye (2023) used APOS to show that procedural actions on hyperbolic functions had not been interiorised into more connected processes and objects. This suggests that instruction should include prediction, explanation, comparison and reflection rather than repeated plotting alone.

Social constructivism adds the importance of guided participation, language and interaction. Learners may develop stronger schemas when they explain why a parabola opens downward, compare two hyperbolas, challenge an incorrect graph or jointly interpret a contextual model. Collaborative work, however, requires careful role design so that learners with disabilities are not assigned passive or clerical roles. Teacher mediation remains essential. The resulting conceptual position combines Universal Design for Learning (UDL), Cognitive Load Theory, APOS theory and social constructivism. UDL foregrounds proactive access, learner agency and multiple means of engagement, representation and expression. Cognitive Load Theory explains why multi-step symbolic work requires careful segmentation and cognitive offloading. APOS distinguishes action-level performance from process, object and schema development, while social constructivism emphasises guided participation, language and collaborative explanation. Figure 1 translates these perspectives into the ACCESS-QH framework. The framework places diagnosis, structured reasoning, representational connection, multimodal access, explicit scaffolding and self-regulation within one inclusive planning cycle. It is presented as an evidence-informed design hypothesis requiring future validation rather than as an already tested intervention.

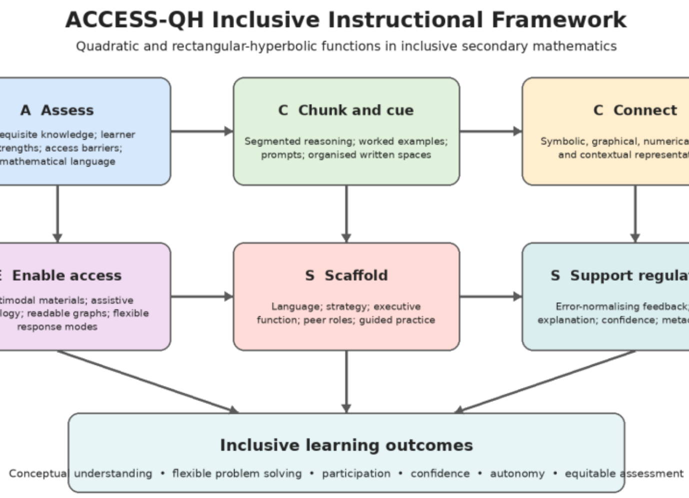

Figure 1. ACCESS-QH inclusive instructional framework for quadratic and rectangular-hyperbolic functions.

The framework shows that inclusive access depends on six connected commitments: assessing prerequisite knowledge and access barriers; chunking and cueing complex reasoning; connecting representations; enabling multimodal and assistive access; scaffolding language, strategy and executive function; and supporting self-regulation and mathematical identity. These commitments converge on conceptual understanding, flexible problem-solving, participation, confidence, autonomy, and equitable assessment.

ACCESS-QH does not prescribe one programme or technology. It provides a planning logic through which teachers, special educators, and support services can select supports that address the actual mathematical and access barriers while maintaining common conceptual goals.

2. Methodology

Design. The study used a critical integrative review design. This approach was selected because the direct empirical literature on disability-specific teaching of quadratic and rectangular hyperbolic functions is sparse, while relevant knowledge is distributed across special education, mathematics education, cognitive psychology, inclusive policy, and technology-enhanced learning. The purpose was not to estimate a single pooled effect or claim exhaustive coverage. It was to integrate complementary evidence, identify transfer limits and produce a defensible instructional framework.

Search process. The review was conducted in July 2026. Searches were undertaken through ERIC, PubMed Central, where educational studies were indexed, DOAJ and publisher journal platforms, supplemented by official South African policy documents and the CAST UDL guidelines. Search strings combined terms such as "quadratic function" OR "quadratic equation"; "hyperbolic function" OR "rectangular hyperbola"; "learning disability" OR "mathematics difficulty" OR "special education"; "Universal Design for Learning"; "cognitive load"; "multiple representations"; "GeoGebra"; "explicit instruction"; "error analysis"; and "inclusive mathematics". Backward reference checking was used to locate relevant reviews and foundational theoretical sources.

Eligibility and evidence boundaries. Priority was given to peer-reviewed work published from 2015 to July 2026 that addressed secondary algebra/functions, mathematics interventions for learners with disabilities or difficulties, UDL in mathematics, or function-specific misconceptions and technology. Earlier sources were retained when they provided a foundational framework, a policy basis, or a frequently used definition. Publications were excluded when they focused solely on tertiary hyperbolic trigonometric functions, provided no educational analysis, or made claims without a traceable scholarly or policy source. Twenty-one core sources were retained for close synthesis: function-specific empirical studies; reviews of algebra or UDL interventions; disability-specific mathematics studies; South African inclusive-education policy analyses; and theoretical sources. Because only a small subset directly studied diagnosed disabilities while teaching these particular functions, disability-specific and general-struggling-learner evidence were coded separately. This distinction prevented findings from general mathematics classrooms from being represented as proven effects for learners with disabilities.

Data extraction and synthesis. Sources were charted by learner group, educational level, topic, design, instructional mechanism, outcome and limitation. The analysis then used a hybrid deductive-inductive process. Deductive categories were derived from UDL, Cognitive Load Theory, and APOS: access, representation, engagement, executive support, action/process/object development, and assessment. Inductive coding identified recurring concerns, including prerequisite gaps, mathematics anxiety, parameter interpretation, asymptotes, reverse reasoning, cognitive offloading, and teacher collaboration.

Quality considerations. Claims were weighted rather than treated as equivalent. Systematic reviews, meta-analyses, quasi-experimental studies, and single-case experimental designs were used to support effectiveness claims within their respective scopes. Case studies were primarily used to understand errors, misconceptions, and contextual processes. Practice-oriented papers informed classroom feasibility but were not treated as controlled evidence. The synthesis also considered contradictory evidence, implementation requirements, and the risk that technology or group work can reproduce exclusion.

Ethical considerations. No human participants, personal data or new empirical dataset were involved in this review. Formal research-ethics approval was therefore not required. Academic integrity was addressed by attributing all evidence, avoiding invented data and explicitly marking the proposed framework as a synthesis requiring future validation.

Table 1. Methodological overview.

Element	Description
Review approach	Critical integrative review bringing together special education, mathematics education, cognitive psychology, inclusive policy and technology-enhanced learning.
Search sources	ERIC, DOAJ, PubMed Central, where educational studies were indexed, publisher journal platforms, official South African policy sources and backward reference checking.
Timeframe	Priority was given to peer-reviewed literature published from 2015 to July 2026; earlier sources were retained when theoretically or historically foundational.
Eligibility	Secondary algebra/functions; mathematics interventions for learners with disabilities or persistent difficulties; UDL; cognitive load; multiple representations; explicit instruction; diagnostic error analysis and accessible technology.
Evidence base	Twenty-one core sources were retained for close synthesis, with disability-specific evidence distinguished from evidence involving broader groups of struggling learners.
Analytical method	Evidence charting by learner group, educational level, topic, design, mechanism, outcome and limitation, followed by hybrid deductive-inductive thematic synthesis.
Ethical boundary	No human participants, personal data or new empirical dataset were involved; no institutional ethics approval was required for the review.

3. Results

The integrative synthesis produced five interrelated themes. The themes combine topic-specific evidence on quadratic and rectangular hyperbolic functions with disability-specific and broader research on mathematics difficulty. Claims are presented within their evidence boundaries; findings from general learner samples are not represented as proven effects for learners with diagnosed disabilities.

Table 2. Theme matrix.

Theme	**Summary finding**	**Inclusive/special education relevance**
1. Cumulative and interactive barriers	Difficulty emerged through interactions among prerequisite gaps, working-memory load, language, anxiety, representation and task design.	Support should target the barrier profile and instructional environment rather than infer global learner inability.
2. Connected representations	Conceptual access improved when equations, tables, graphs, verbal descriptions and contexts were explicitly linked.	Multiple representations require accessible design, verbal description and opportunities to translate among forms.
3. Explicit instruction and cognitive offloading	Systematic modelling, organised written spaces, worked examples, cueing and gradual fading reduced unnecessary load.	Scaffolds can provide low-stigma access while preserving the same conceptual goals and mathematical standards.
4. Diagnostic use of errors	Errors in roots, parameter effects, asymptotes and notation often revealed deeper misconceptions rather than carelessness.	Feedback should identify the relationship needing revision and use error analysis to guide individualised support.
5. Flexible language, participation and assessment	Vocabulary, visual density, response mode and executive demands shaped who could participate and demonstrate understanding.	Assessment should preserve the construct while reducing irrelevant barriers related to sight, handwriting speed, reading load or memory.

3.1 Theme 1: Barriers are cumulative and interactive

The evidence indicates that poor performance cannot be explained by a single misconception or disability label. Quadratic and hyperbolic tasks accumulate demands. A learner may need to recall integer rules, interpret function notation, coordinate axes, select a method, maintain intermediate results, read mathematical English and monitor accuracy. Difficulty in any one component can consume resources needed for the others. Mathematics anxiety further competes for attention and working memory; the meta-analysis by Barroso et al. (2021) found a consistent negative association between mathematics anxiety and achievement.

Prerequisite gaps are especially consequential. Learners who cannot flexibly expand brackets, work with negative numbers, factorise, solve linear equations or interpret fractions face a double task: learning the function concept while reconstructing earlier knowledge. Inclusive instruction should therefore begin with low-stakes prerequisite assessment and rapid, targeted reteaching rather than assuming that repeated exposure to the new topic will repair the gap.

3.2 Theme 2: Conceptual access depends on connected representations

Across the function-specific studies, a central problem was representational discontinuity. Learners could manipulate an equation without seeing its graph, plot points without understanding parameter effects, or identify a visual feature without expressing it symbolically. Quadratic teaching is more accessible when standard, factored and vertex forms are connected to y-intercepts, roots and turning points. Hyperbolic teaching is more accessible when the denominator restriction, vertical asymptote, horizontal shift and branch location are coordinated rather than taught as separate rules.

Dynamic graphing can make these relationships visible. Sebsibe and Abdella (2025) compared 42 learners receiving GeoGebra-integrated instruction with 45 learners receiving traditional instruction. The experimental group showed significantly stronger conceptual and problem-solving performance, and 40 of 42 reported motivation to learn the topic. Technology is therefore promising, but its inclusive value depends on accessible interfaces, keyboard navigation, readable labels, non-colour cues, teacher explanation and an offline alternative.

3.3 Theme 3: Explicit instruction and cognitive offloading support participation

Explicit instruction is most effective when it makes thinking visible rather than merely prescribing steps. For a quadratic problem, the teacher can model how to classify the equation, identify possible methods, organise factor pairs, check the middle term, interpret solutions and verify against a graph. For a hyperbolic function, the teacher can model how to identify asymptotes, choose test points, predict branches and check intercepts. Prompts can then be faded from full worked examples to partially completed examples, and then to independent practice.

External organisation reduces unnecessary working-memory demands. Dedicated boxes for coefficients, factor pairs and checks; one line per transformation; labelled asymptote fields; and a representation-matching table can prevent visual clutter. These supports are consistent with Hord et al. (2025) and the broader algebra-intervention evidence. They should be available to all learners while remaining customisable for those needing more intensive support.

3.4 Theme 4: Errors should be diagnosed, interpreted and used instructionally

Error analysis was a strong bridge between mathematics education and special education. Makgakga (2023) showed that an answer may look procedural while concealing an incorrect meaning of "root." Matindike and Makonye (2023) demonstrated that visible errors in graphing or notation can reflect deeper misconceptions about asymptotes and the structure of functions. Teachers, therefore, need diagnostic questions that reveal reasoning: "What does this root represent on the graph?", "Why can x not equal this value?", "What changes and what remains invariant when p changes?"

Feedback should identify the mathematical relationship requiring revision, not simply mark the response wrong. Learners can compare an incorrect worked example with a correct one, explain the first point of divergence and repair it. This approach positions error as information and can reduce shame, especially for learners with histories of repeated failure.

3.5 Theme 5: Access requires flexible language, participation and assessment

Function learning has substantial language demands: increasing, decreasing, axis, intercept, turning point, asymptote, domain, range, undefined, reciprocal and transformation. Teachers should preteach vocabulary, pair terms with diagrams and examples, permit initial explanation in a familiar language where possible, and distinguish everyday from mathematical meanings. Visual displays should be accompanied by verbal descriptions for learners who cannot rely solely on sight.

Assessment should preserve the construct while reducing irrelevant barriers. A learner may demonstrate understanding by annotating a graph, orally explaining parameter effects, completing a structured table, using an accessible graphing tool or writing a symbolic solution. Flexibility does not mean accepting a lower mathematical standard; it means ensuring that handwriting speed, visual acuity, reading load, or memory for lengthy instructions are not inadvertently made the unintended object of assessment. Nieminen (2022) similarly argues for a universally designed mathematics assessment that anticipates learner variability.

Function-specific inclusive responses include labelled workspaces for factor pairs and asymptotes; side-by-side equation, table and graph displays; high-contrast and colour-independent graph conventions; tactile or enlarged graph options; vocabulary preteaching; bilingual discussion where feasible; decision trees for method selection; extended time; segmented instructions; and accessible graphing tools. These supports maintain the mathematical construct while reducing demands that are incidental to it.

4. Discussion

4.1 Accessible teaching as layered inclusive scaffolding

The review supports a layered account of inclusion. Learners do not gain access to quadratic and hyperbolic functions through a single method, device or accommodation. Access emerges when the mathematical structure, the learner's current knowledge, the representation system, the language of instruction, the task layout, the emotional climate and the assessment format are aligned. This explains why a strategy may be effective in one lesson but insufficient in another. The strongest convergence occurs around explicit structure plus conceptual connection. Special-education reviews favour systematic modelling, guided practice and feedback; function-specific studies reveal the cost of procedural teaching disconnected from meaning. The appropriate synthesis is therefore not "direct instruction versus discovery." Teachers can provide explicit initial models and then create carefully scaffolded opportunities for predicting, comparing, explaining, and investigating. Learners should not be left to discover inaccessible structures unaided, but neither should they be confined to imitating procedures.

The strongest convergence occurs around explicit structure combined with conceptual connection. Special-education reviews support systematic modelling, guided practice and corrective feedback, while function-specific studies expose the weaknesses of procedures detached from meaning. Accessible teaching should therefore begin with explicit models and progress towards prediction, comparison, explanation, investigation and transfer. Learners should not be left to discover inaccessible structures without support, but neither should they be confined to imitating steps.

4.2 Risks of exclusion and evidence boundaries

The review also shows why topic-specific special-education research is needed. Evidence supporting UDL, explicit algebra instruction and dynamic software is encouraging, yet few studies directly examine quadratic or rectangular-hyperbolic functions with well-described disability groups. Generalising from struggling learners to diagnosed learning disabilities must therefore remain cautious. Future work should report learner characteristics, accommodations, intervention fidelity, teacher expertise, accessible technology features and both conceptual and procedural outcomes.

The South African context adds material and linguistic constraints. Dynamic graphing is promising, but schools may have limited devices, data, electricity or technical support. Inclusive design should therefore include printable graph sequences, teacher-projected demonstrations, shared-device routines and offline software. Equally, instruction must recognise multilingual classrooms. Learners' informal or home-language explanations can serve as bridges to formal terminology rather than being treated as evidence of mathematical weakness. Dynamic graphing and assistive technologies are promising, but technology is not inherently inclusive. Interfaces may be inaccessible to screen readers, graphs may rely solely on colour, labels may be announced in an unusable sequence, and learners may lack devices, data, electricity, or technical support. Every technology-supported lesson, therefore, requires an accessible low-tech or offline pathway and active teacher mediation.

4.3 Implications for practice

Teach fewer relationships more explicitly before increasing task variety. A quadratic lesson should not present factorisation, roots, graphs, and modelling as unrelated procedures. Select one structural relationship, for example, roots as x-intercepts, and revisit it across representations. A hyperbolic lesson can similarly centre the relationship between the denominator restriction and the vertical asymptote.

Use a predictable lesson architecture: prerequisite retrieval; explicit goal and vocabulary; teacher modelling with think-aloud; guided comparison of examples and non-examples; supported practice; representation translation; independent application; and a brief exit diagnostic. Predictability reduces executive demands while leaving room for rich reasoning.

Plan visual accessibility. Do not rely only on red and green curves. Use labels, line patterns, thick axes and high contrast. Describe graphs aloud, state the scale, mark key points and ensure that printed coordinates remain legible. Provide tactile or raised-line options where required and consult specialists rather than assuming that a digital graph is automatically accessible.

Use diagnostic feedback. Record common error types and design short response tasks around them. For example, present a parabola whose roots are correct, but the turning point is wrong, or a hyperbola whose asymptotes are correct but the branches are misplaced. Ask learners to identify, explain and repair the error.

Special educators should participate in planning the mathematical representation, not only simplify worksheets after they have been created. Their expertise in task analysis, prompting, assistive technology, behaviour support and progress monitoring can be combined with the mathematics teacher's content knowledge.

Co-teaching roles should vary. One teacher can model while the other records misconceptions; parallel groups can provide different pacing without different standards; and station teaching can separate prerequisite repair, dynamic graph exploration and teacher-led conceptual discussion. Support should remain visible as part of normal classroom practice, so that individual learners are not stigmatised.

Individualised supports should specify the barrier and the intended mathematical access. "Use a calculator" is too vague. A more useful plan identifies whether the learner needs reduced arithmetic load, speech output, enlarged keys, equation entry support or time to check signs. Progress monitoring should examine independence, explanation and transfer, not only the number of correct answers.

Pre-service and in-service programmes should include advanced-content inclusion. Training often demonstrates UDL with simple tasks, leaving teachers uncertain about how to preserve rigour in algebra. Teachers need opportunities to redesign real quadratic and hyperbolic lessons, test accessible graphing tools, analyse learners' work, and rehearse explanations of roots, vertices, asymptotes, and parameter transformations.

Professional learning should integrate knowledge of disability with pedagogical content knowledge. Teachers need to distinguish between a misconception and a transcription error, a language barrier and a conceptual barrier, and accommodation and a change in the construct assessed. Collaborative lesson study involving mathematics teachers, special educators and learning-support staff can build this expertise.

In practical terms, a lesson sequence should begin with a brief prerequisite probe, make the learning goal and vocabulary explicit, model one mathematically meaningful phase at a time, provide guided comparison of examples and non-examples, connect representations, and end with a short diagnostic task. Special educators and mathematics teachers should co-plan the representation and support strategy, not merely simplify a worksheet after it has been designed. Progress monitoring should examine explanation, independence and transfer in addition to correct answers.

4.4 Implications for policy and school leadership

School leaders should allocate time for co-planning and ensure accessibility is built into procurement. Graphing software, projectors, printers and assistive devices require maintenance, training and offline contingencies. A school that purchases software without accessible interfaces or teacher preparation may widen rather than reduce exclusion.

Support teams should use assessment data to organise tiered assistance. Whole-class UDL can be combined with short targeted groups and intensive individual intervention. Referral procedures should not require a learner to fail repeatedly before receiving structured support. Psychological, visual, hearing, and occupational therapy services should be coordinated with classroom instruction, where available.

Assessment specifications should state the mathematical construct clearly and remove avoidable barriers. Graphs should be legible, instructions segmented and vocabulary consistent with the instructions. Alternative formats should be prepared in advance. Where technology is allowed for learning, the assessment policy should clarify whether and how accessible graphing tools may be used.

Curriculum guidance should include misconception profiles, accessible examples and differentiated pathways that maintain common conceptual goals. Policy documents should avoid equating inclusion with physical placement alone. Participation in higher-level mathematical reasoning is a substantive measure of inclusion.

School and district support should connect inclusive education, curriculum access and digital procurement. Time for co-planning, maintenance of graphing and assistive technologies, accessible assessment formats, tiered support pathways and sustained professional learning are not peripheral conditions; they determine whether inclusive instruction can be implemented consistently. Curriculum guidance should include misconception profiles, accessible examples and differentiated pathways that maintain common conceptual goals.

4.5 Limitations and future research

This integrative review has four limitations. First, it is focused rather than exhaustive and was not registered as a systematic review. Second, direct disability-specific research on quadratic and rectangular-hyperbolic functions is scarce, so parts of the framework draw on evidence from broader algebra, UDL, and mathematics-difficulty studies. Third, the available studies vary considerably in terms of learner age, disability status, and curriculum and design, preventing a pooled estimate of effectiveness. Fourth, several topic-specific studies are situated in Southern African contexts; this is a strength for contextual relevance but limits universal generalisation.

The proposed framework should therefore be treated as a theoretically and empirically informed design hypothesis. Its individual components have support, but the combined framework requires validation, adaptation and fidelity study.

Future research should test ACCESS-QH through design-based and quasi-experimental studies in inclusive secondary classrooms. Studies should recruit and describe learners with specific learning disabilities, intellectual disabilities, autism, attention-related needs, visual impairments and language-related barriers rather than using an undifferentiated "special needs" category. Where samples are small, rigorous single-case designs can establish functional relations.

Outcomes should include conceptual understanding, procedural fluency, representation translation, strategy selection, mathematics anxiety, participation and independence. Researchers should report whether learners can generalise from a supported example to a novel quadratic or hyperbolic task. Technology studies should evaluate accessibility features and not merely compare devices with no devices.

Participatory research is also needed. Learners with disabilities can identify barriers that are invisible to researchers, such as graph labels that screen readers announce in an unusable order or worksheets whose visual density triggers disengagement. Teachers should be involved in designing feasible interventions for low-resource and multilingual contexts.

Future research should test the ACCESS-QH framework through design-based, quasi-experimental and rigorous single-case studies in inclusive secondary classrooms. Studies should describe learner characteristics and accommodations precisely, evaluate intervention fidelity, and report conceptual understanding, procedural fluency, representation translation, strategy selection, anxiety, participation, independence and generalisation.

5. Conclusion

Quadratic and rectangular-hyperbolic functions should not become gatekeeping topics that exclude learners from advanced mathematics. Their complexity is real, but exclusion is not inevitable. The evidence indicates that learners benefit when teachers diagnose prerequisites, structure multi-step work, connect representations, use technology purposefully, teach language explicitly, analyse errors and provide flexible ways to participate and demonstrate understanding.

The ACCESS-QH framework converts these principles into six connected planning commitments: Assess, Chunk and cue, Connect, Enable access, Scaffold, and Support self-regulation. Its central proposition is that inclusive education does not dilute the demand for mathematics. It removes demands that are irrelevant to the intended concept while strengthening the representations, explanations, strategies and relationships through which rigorous understanding becomes possible.

For the International Journal of Special Education, the contribution is twofold. It brings advanced function content into a literature that often concentrates on foundational numeracy, and it offers a research agenda that joins special-education methodology with mathematics pedagogical content knowledge. Equitable participation in quadratic and hyperbolic reasoning is not an optional extension of inclusion; it is part of the right to a full mathematics curriculum.

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- **Ethics approval and consent to participate:** This integrative review involved no human participants, personal data or new empirical fieldwork; institutional ethics approval and participant consent were therefore not required.
- **Studies involving animals:** This review did not involve animals.
- **Data availability:** No new dataset was generated. The evidence sources used in the synthesis are listed in the reference section.
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