Algebraic Operations and Topological Framework on Order (k) and Degree (j) Sets

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ABSTRACT:
The operations such as union, intersection, complement are extended to proposed (k, j)-picture sets. These operations play an important role in developing the theoretical framework of (k, j)-picture set theory. The fundamental properties that exist in classical set theory are also examined within this new setting. By studying these properties, similarities and differences between classical sets and (k, j)-picture sets can be identified. The behaviour of these operations helps in understanding the structure of (k, j)-picture sets more clearly. In particular, they lead to introduce the topological structure on a family of (k, j)-picture sets. This topology is defined over a non-empty universe X and helps to study continuity and related concepts in the context of (k, j)-picture set theory.

Keywords: (k,j)-picture sets, union, intersection, complement, (k,j)-picture topology.

1. Introduction

Classical logic statements assigns a value of either true or false; however, this binary framework is inadequate for modelling uncertainty encountered in real-world situations. To address this limitation, Zadeh [14] introduced Fuzzy set theory, where each element is characterized by a membership degree in the interval [0,1]. Since numerous extensions have been proposed, including Intuitionistic fuzzy sets [1], Soft sets [7], Fuzzy soft sets [2], and Rough sets [9], along with other models such as Vague soft sets [13] and Intuitionistic multi-fuzzy sets [8]. Furthermore, Neutrosophic sets [12] provide a more general framework for handling uncertainty, indeterminacy, and inconsistency.

Among these developments, Picture fuzzy sets (PFS), introduced by Cuong [4], extend Intuitionistic fuzzy sets by incorporating neutral and refusal membership degrees in addition to membership and non-membership. This enables a more flexible representation of uncertainty in situations involving multiple viewpoints, such as decision-making and voting systems. Consequently, PFS have been widely applied in areas such as material selection, pattern recognition, and algebraic structures [6]. Moreover, beyond their practical relevance, fuzzy and related set theories also play a significant role in theoretical studies, particularly in the development of fuzzy topological spaces and their generalizations.

2. Preliminaries

We begin by introducing some basic notions. First, we recall the fundamental concepts of fuzzy sets and Intuitionistic fuzzy sets. These will then be compared with Picture fuzzy sets and Picture sets. In addition, Intuitionistic sets are also briefly discussed.

Definition 2.1. [14] Let X be a non-empty universal set. A fuzzy set M on X is an object of the form: $M = \{(x, \mu_M(x)); x \in X\}$ where $\mu_M(x): X \to [0, 1]$ is a function that assigns a real number from the interval $[0, 1]$ to each element of X. The value $\mu_M(x)$ shows the grade of membership of x in M.

Definition 2.2. [1] An Intuitionistic fuzzy set (IFS) M on a non-empty set X is an object of the form $M = \{(x, \mu_M(x), \nu_M(x)); x \in X)\}$ where $\mu_M(x) \in [0, 1]$ is called the degree of membership of x in M, $\nu_M(x) \in [0, 1]$ is called the degree of non-membership of x in M and we assume that $\mu_M(x)$ and $\nu_M(x)$ satisfy the following condition $\forall x \in X$ $(\mu_M(x) + \nu_M(x) \leq 1)$

Definition 2.3. [3], Let X be a non-empty universal set. We say that $M \subseteq X$ is an Intuitionistic set (IS) if M is of the form $M = \langle X, M_1, M_2 \rangle$, where M_1 and M_2 are the subsets of X such that $M_1 \cap M_2 = \emptyset$. The sets M_1 and M_2 are interpreted as members and non-members of M respectively.

Definition 2.4. [3], Let X be a non-empty set, $M = \langle X, M_1, M_2 \rangle$ and $N = \langle X, N_1, N_2 \rangle$ be two Intuitionistic sets (IS) on X. Let J be a non-empty index set and $\{M_i : i \in J\}$ be an arbitrary family of IS's on X, where each of its elements is of the form $M_i = \langle X, M^{(1)}, M^{(2)} \rangle$ for any $i \in J$.

Definition 2.5. [4] Let X be a non-empty universe. A Picture fuzzy set (PFS) M on a non-empty universe X is an object of the form $M=\{(x, \mu_M(x), \eta_M(x), \nu_M(x)): x \in X\}$ where $\mu_M(x) \in [0, 1]$ is called the grade of positive membership of x in M, $\eta_M(x) \in [0, 1]$ is called the degree of neutral membership of x in M, $\nu_M(x) \in [0, 1]$ is called the degree of negative membership of $x \in M$. We assume that $\mu_M(x) + \eta_M(x) + \nu_M(x) \leq 1$ for any $x \in X$. Then $1-(\mu_M(x) + \eta_M(x) + \nu_M(x))$ is called the degree of refusal membership of x in M.

Definition 2.6. [5] Assume that X is a non-empty universe. A Picture set on X is an object of the form $M = \langle X, M_1, M_2, M_3 \rangle$ where M_1, M_2 and M_3 are the subsets of X such that $M_1 \cap M_3 = \emptyset$. The sets M_1, M_2, M_3 and $(M_1 \cup M_2 \cup M_3)^c$ are the sets of members, neutral members, non-members and refusal members of M respectively.

Note that in Neutrosophic crisp sets we have stronger assumptions that $M_1 \cap M_2 = \emptyset$, $M_1 \cap M_3 = \emptyset$ and $M_2 \cap M_3 = \emptyset$ [10, 11]. Hence, each neutrosophic crisp set can be considered as a picture set but the converse of this statement is not true.

Definition 2.7. [5] Let X be a non-empty universe. A collection τ_δ of picture subsets on X is called Picture topology on X if and only if the following axioms are satisfied:

1. $X_\delta, \emptyset_\delta \in \tau_\delta$.
2. τ_δ is closed under arbitrary picture unions and finite picture intersections.

An ordered pair (X, τ_δ) is called Picture topological space. Members of τ_δ are known as picture open sets ($P_\delta OS$) in X. Their picture complements are picture closed sets ($P_\delta CS$).

3. Operators on order (k) degree (j) Picture sets

3.1 Union and intersection on order (k) and degree (j) Picture sets

Let L, M and N be any three (k,j)-picture sets that are associated with set valued functions P_L, P_M and P_N respectively so that $L=\langle X:P_L \rangle$, $M=\langle X:P_M \rangle$ and $N=\langle X:P_N \rangle$. Now the following expressions hold for L and M.

$$(L_1 \cup M_1) \cap (L_j \cap M_j) = L_1 \cap (L_j \cap M_j) \cup M_1 \cap (L_j \cap M_j)$$

$$\subseteq (L_1 \cap L_j) \cup (M_1 \cap M_j) = \emptyset \cup \emptyset = \emptyset \quad (3.1)$$

$$(L_1 \cap M_1) \cap (L_j \cup M_j) = (L_1 \cap M_1) \cap L_j \cup (L_1 \cap M_1) \cap M_j$$

$\subseteq (L_1 \cap L_j) \cup (M_1 \cap M_j) = \varnothing \cup \varnothing = \varnothing$. (3.2)

The above two expressions (3.1) and (3.2) motivate us to define $^{(k,j)}\bigcap_{ps}$ and $^{(k,j)}\bigcup_{ps}$ on (k,j)-picture sets as given in the next definition.

Definition 3.1.1:

(i) $L\,^{(k,j)}\bigcup_{ps} M = \langle X:(L_1 \cup M_1, L_2 \cap M_2, L_3 \cap M_3, \ldots L_k \cap M_k)\rangle$

(ii) $L\,^{(k,j)}\bigcap_{ps} M = \langle X:(L_1 \cap M_1, L_2 \cup M_2, L_3 \cup M_3, \ldots L_k \cup M_k)\rangle$

Definition 3.1.2: Let $[1,k] \cap Z = \{1,2,3\ldots k\}$ be a digital interval. If $f:[1,k] \cap Z \to 2^X$ and $g: [1,k] \cap Z \to 2^X$ are set valued functions from the digital interval $[1,k] \cap Z$ to the subsets of X then $f \vee g: [1,k] \cap Z \to 2^X$ and $f \wedge g: [1,k] \cap Z \to 2^X$ are defined as $(f \vee g)(1) = f(1) \cup g(1)$, $(f \vee g)(i) = f(i) \cap g(i)$ and $(f \wedge g)(1) = f(1) \cap g(1)$,

$(f \wedge g)(i) = f(i) \cup g(i)$ for $i=2,3,4,\ldots k$.

Proposition 3.1.3: Let $L=\langle X:P_L\rangle$, $M=\langle X:P_M\rangle$.

(i) $L\,^{(k,j)}\bigcup_{ps} M = \langle X:P_L \vee P_M\rangle$.

(ii) $L\,^{(k,j)}\bigcap_{ps} M = \langle X:P_L \wedge P_M\rangle$.

Proof: Let $L=\langle X:P_L\rangle$, $M=\langle X:P_M\rangle$ where $P_L(1) = L_1$, $P_L(i) = L_i$ and $P_M(1) = M_1$, $P_M(i) = M_i$ for $i=2,3,..,k$.

$(P_L \vee P_M)(1) = P_L(1) \cup P_M(1) = L_1 \cup M_1$ and $(P_L \vee P_M)(i) = P_L(i) \cap P_M(i) = L_i \cap M_i$. This proves (i).

$(P_L \wedge P_M)(1) = P_L(1) \cap P_M(1) = L_1 \cap M_1$ and $(P_L \wedge P_M)(i) = P_L(i) \cup P_M(i) = L_i \cup M_i$. This proves (ii).

Proposition 3.1.4: If $L,M \in PS[X:(k,j)]$ then $L\,^{(k,j)}\bigcup_{ps} M \in PS[X:(k,j)]$, $L\,^{(k,j)}\bigcap_{ps} M \in PS[X:(k,j)]$,

Proof: Follows from (3.1), (3.2), Definition 3.1.1 and Proposition 3.1.3.

Lemma 3.1.5: Let $L,M,N \in PS[X:(k,j)]$ such that $L\,^{(k,j)}\subseteq_{ps} N$ and $M\,^{(k,j)}\subseteq_{ps} N$. Then

(i) $L\,^{(k,j)}\bigcup_{ps} M\,^{(k,j)}\subseteq_{ps} N$.

(ii) $L\,^{(k,j)}\bigcap_{ps} M\,^{(k,j)}\subseteq_{ps} N$.

Proof: Suppose $L\,^{(k,j)}\subseteq_{ps} N$ and $M\,^{(k,j)}\subseteq_{ps} N$.

$L\,^{(k,j)}\subseteq_{ps} N \Rightarrow P_L(1) \subseteq P_N(1)$, $P_L(i) \supseteq P_N(i)$ for $i=2,3,4\ldots k$ (3.3)

$M\,^{(k,j)}\subseteq_{ps} N \Rightarrow P_M(1) \subseteq P_N(1)$, $P_M(i) \supseteq P_N(i)$ for $i=2,3,4\ldots k$ (3.4)

Using (3.3) and (3.4) we have

$P_L(1) \cup P_M(1) \subseteq P_N(1)$, $P_L(i) \cap P_M(i) \supseteq P_N(i)$ for $i=2,3,4\ldots k$ that implies

$(P_L \vee P_M)(1) \subseteq P_N(1)$, $(P_L \vee P_M)(i) \supseteq P_N(i)$ for $i=2,3,4\ldots k$

Therefore $L\,^{(k,j)}\bigcup_{ps} M\,^{(k,j)}\subseteq_{ps} N$.

Again using (3.3) and (3.4) we have

$P_L(1) \cap P_M(1) \subseteq P_N(1)$, $P_L(i) \cup P_M(i) \supseteq P_N(i)$ for $i=2,3,4\ldots k$ that implies

$(P_L \wedge P_M)(1) \subseteq P_N(1)$, $(P_L \wedge P_M)(i) \supseteq P_N(i)$ for $i=2,3,4\ldots k$

Therefore $L\,^{(k,j)}\bigcap_{ps} M\,^{(k,j)}\subseteq_{ps} N$.

3.2 Complements of order (k) and Degree (j) Picture sets

Let $P_{\varsigma L}$ be a set function associated with ςL, the complement of L. The complement ςL of L should be defined in such a way that

$L\,^{(k,j)}\bigcap_{ps} \varsigma L = [\varnothing]_{(k,j)}$, (3.5)

$L\,^{(k,j)}\bigcup_{ps} \varsigma L = [X]_{(k,j)}$ (3.6)

Let $M = \varsigma L$.

$L\,^{(k,j)}\bigcap_{ps} M = [\varnothing]_{(k,j)} \Leftrightarrow L_1 \cap M_1 = \varnothing$, $L_i \cup M_i = X$ for $i=2,3,\ldots,k$ (3.7)

$L\,^{(k,j)}\bigcup_{ps} M = [X]_{(k,j)} \Leftrightarrow L_1 \cup M_1 = X$ and $L_i \cap M_i = \varnothing$ for $i=2,3,\ldots,k$ (3.8)

Therefore (3.5) and (3.6) hold $\Leftrightarrow$ (3.7) and (3.8) hold.

$\Leftrightarrow (L_1 \cap M_1 = \varnothing, L_1 \cup M_1 = X)$ and $(L_i \cup M_i = X, L_i \cap M_i = \varnothing)$, for $i=2,3,4,\ldots k$

$\Leftrightarrow$ $M_1 = X \setminus L_1$, $M_i = X \setminus L_i$ for $i=2,3,\dots k$

Therefore it is expected to choose M as the complement of L we wish to take

$M_1 = X \setminus L_1$, $M_j = X \setminus L_j$ and $M_i = X \setminus L_i$ for $i \neq 1$ and j. (3.9)

Since M should be a (k,j)-picture set, what we need is $M_1 \cap M_j = \varnothing$ that is

$(X \setminus L_1) \cap (X \setminus L_j) = \varnothing$ that implies $X \setminus (L_1 \cup L_j) = \varnothing$ so that $L_1 \cup L_j = X$

The only property of the (k,j)-picture set is $L_1 \cap L_j = \varnothing$ so that $L_j \subseteq X \setminus L_1$, $L_1 \subseteq X \setminus L_j$. This together with (3.9) we have $L_j \subseteq X \setminus L_1$, $L_1 \subseteq X \setminus L_j$, $M_1 = X \setminus L_1$, $M_j = X \setminus L_j$. Since M_1 and L_j are both subsets of $X \setminus L_1$ and also M_j and L_1 are both subsets of $X \setminus L_j$, the only choice to define the complement of L is that $M_1 = L_j$ and $M_j = L_1$. Thus we have following definition for the complement of L.

Definition 3.2.1: Let $P_{\varsigma L}:\{1,2,3,\dots,k\} \to 2^X$, be defined as $P_{\varsigma L}(1) = P_L(j)$, $P_{\varsigma L}(j) = P_L(1)$ and

$P_{\varsigma L}(i) = X \setminus P_L(i)$ $i \neq 1, i \neq j$. Then the complement of L is defined as $\varsigma L = \langle X: P_{\varsigma L} \rangle$.

Proposition 3.2.2: $L \in PS[X:(k,j)] \Leftrightarrow \varsigma L \in PS[X:(k,j)]$.

Proof: Let $L = \langle X: P_L \rangle$ be a (k,j)-picture set on X. Then $\varsigma L = \langle X: P_{\varsigma L} \rangle$. Now $P_{\varsigma L}(1) = P_L(j)$,

$P_{\varsigma L}(j) = P_L(1)$ and $P_{\varsigma L}(i) = X \setminus P_L(i)$ where $i \neq 1$ and j so that $P_{\varsigma L}(1) \cap P_{\varsigma L}(j) = P_L(j) \cap P_L(1) = \varnothing$. Therefore $\varsigma L = \langle X: P_{\varsigma L} \rangle$ is a (k,.j)-picture on X.

Conversely, using if $L \in PS[X:(k,j)]$ then $\varsigma L \in PS[X:(k,j)]$

$$\Rightarrow \varsigma(\varsigma L) \in PS[X:(k,j)]$$

$$\Rightarrow \in PS[X:(k,j)]$$

4 Order (k) and Degree (j) Picture Topology

Definition 4.1: Let X be a non empty universe. Let j, k be any two integers such that $2 \leq j \leq k$. A collection $\mathcal{P}$ of (k,j)-picture sets on X is said to be a (k,j)-picture Topology on X if the following axioms hold.

(i) $[\varnothing]_{(k,j)} \in \mathcal{P}$, $[X]_{(k,j)} \in \mathcal{P}$,

(ii) $L \ {}^{(k,j)}\!\cap_{ps} M \in \mathcal{P}$ whenever $\{L, M\} \subseteq \mathcal{P}$,

(iii) ${}^{(k,j)}\!\cup_{ps}\{M_\alpha : \alpha \in \Omega$, where Ω is a index set$\} \in \mathcal{P}$ for every arbitrary collection

$\{M_\alpha \in PS[X:(k,j)], \alpha \in \Omega\} \subseteq \mathcal{P}$

If $\mathcal{P}$ is a (k,j)-Picture topology on X, the ordered pair $(X, \mathcal{P})$ is a (k,j)-Picture Topological space on X and members of $\mathcal{P}$ are known as (k,j)-picture open sets in $(X, \mathcal{P})$. The (k,j)-complement of a (k,j)-picture open set is (k,j)-picture closed set in $(X, \mathcal{P})$. The family all (k,j)-picture closed set in $(X, \mathcal{P})$ is denoted by $\varsigma\mathcal{P}$. Therefore if M is a (k,j)-picture open set in $(X, \mathcal{P})$ then ςM is (k,j)-picture closed set in $(X,\mathcal{P})$ and belongs to $\varsigma\mathcal{P}$.

Proposition 4.2: Let $(X, \mathcal{P})$ be a (k,j)-Picture Topological space then the following three conditions holds for closed sets.

(i) $[\varnothing]_{(k,j)} \in \varsigma\mathcal{P}$, $[X]_{(k,j)} \in \varsigma\mathcal{P}$,

(ii) If $M, N \in \varsigma\mathcal{P}$ then $M \ {}^{(k,j)}\!\cup_{ps} N \in \varsigma\mathcal{P}$,

(iii) If $\{M_\alpha \in PS[X: (k,j)] : \alpha \in \Omega\} \subseteq \varsigma\mathcal{P}$ then ${}^{(k,j)}\!\cap_{ps}\{M_\alpha : \alpha \in \Omega\} \in \varsigma\mathcal{P}$

Proof: we see that $[\varnothing]_{(k,j)} = \varsigma[X]_{(k,j)} \in \varsigma\mathcal{P}$and

$[X]_{(k,j)} = \varsigma[\varnothing]_{(k,j)} \in \varsigma\mathcal{P}$. This proves (i).

Now let $M, N \in \varsigma\mathcal{P}$. Then $\varsigma M \in \mathcal{P}$ and $\varsigma N \in \mathcal{P}$.

By using Definition 4.1(ii) we have $\varsigma M \ {}^{(k,j)}\!\cap_{ps} \varsigma N \in \mathcal{P}$, then $\varsigma M \ {}^{(k,j)}\!\cap_{ps} \varsigma N = \varsigma(M \ {}^{(k,j)}\!\cup_{ps} N)$

that implies $M \ {}^{(k,j)}\!\cup_{ps} N \in \varsigma\mathcal{P}$. This proves (ii).

Now let $\{M_\alpha \in PS[X:(k,j)] : \alpha \in \Omega\} \subseteq \varsigma\mathcal{P}$. This implies $\{\varsigma M_\alpha \in PS[X:(k,j)] : \alpha \in \Omega\} \subseteq \mathcal{P}$.

Therefore by using Definition 4.1(iii), we have ${}^{(k,j)}\!\cup_{ps}\{\varsigma M_\alpha : \alpha \in \Omega\} \in \mathcal{P}$ that implies,

$\varsigma\left({}^{(k,j)}\!\cap_{ps}\{M_\alpha : \alpha \in \Omega\}\right) = {}^{(k,j)}\!\cup_{ps}\{\varsigma M_\alpha : \alpha \in \Omega\} \in \mathcal{P}$ so that ${}^{(k,j)}\!\cap_{ps}\{M_\alpha : \alpha \in \Omega\} \in \varsigma\mathcal{P}$.

This proves (iii).

Proposition 4.3: $\mathcal{Q}$ is a collection (k,j)-picture sets on X satisfying the following three conditions then $\varsigma\mathcal{Q} = \{\varsigma M : M \in \mathcal{Q}\}$ is a (k,j)-Picture Topology on X.

(i) $[\varnothing]_{(k,j)} \in \mathcal{Q}$, $[X] \in \mathcal{Q}$,

(ii) If M , N $\in \mathcal{Q}$ then $M\,{}^{(k,j)}\bigcup_{ps} N \in \mathcal{Q}$,

(iii) If $\{M_\alpha \in PS[X:(k,j)] : \alpha \in \Omega\} \subseteq \mathcal{Q}$ then ${}^{(k,j)}\bigcap_{ps}\{M_\alpha : \alpha \in \Omega\} \in \mathcal{Q}$.

Proof: Analogous to Proposition 4.2.

Remark 4.4: A (k,j)-picture set M is (k,j)-picture closed set in (X, $\mathcal{P}$) iff the (k,j)-picture complement of M is (k,j)-picture open set in (X, $\mathcal{P}$).

Definition 4.5: Let $\mathcal{P}$ and $\mathcal{R}$ be any two (k,j)-Picture Topologies on X. If $\mathcal{P} \subseteq \mathcal{R}$ then $\mathcal{P}$ is (k,j)-coarser than $\mathcal{R}$ and $\mathcal{R}$ is (k,j)-finer than $\mathcal{P}$. If $\mathcal{P}$ is neither (k,j)- coarser than $\mathcal{R}$ nor (k,j)-finer than $\mathcal{R}$ then $\mathcal{P}$ and $\mathcal{R}$ are said to be not comparable.

Examples 4.6: Let X be non empty universe. Then $\{[\varnothing]_{(k,j)}, [X]_{(k,j)}\}$ and PS[X:(k,j)]are respectively the (k,j)-coarset and (k,j)-finest Picture Topologies in the family of all (k,j)-Picture Topologies on X. These topologies are respectively called the (k,j)-indiscrete picture topology and the (k,j)-discrete picture topology on X.

Examples 4.7: Let $\mathbb{R}$ denote the set of all real numbers. Let M, N be any two (k,j)-picture sets that are not (k,j)-comparable on $\mathbb{R}$. Let

$$\mathcal{P} = \{[\varnothing]_{(k,j)}, [\mathbb{R}]_{(k,j)}, M, N, M\,{}^{(k,j)}\bigcap_{ps} N, M\,{}^{(k,j)}\bigcup_{ps} N\}$$

$$\mathcal{Q} = \{[\varnothing]_{(k,j)}, [\mathbb{R}]_{(k,j)}, M, M\,{}^{(k,j)}\bigcap_{ps} N\}$$

$$\mathcal{R} = \{[\varnothing]_{(k,j)}, [\mathbb{R}]_{(k,j)}, N, M\,{}^{(k,j)}\bigcap_{ps} N\}$$

$$\mathcal{S} = \{[\varnothing]_{(k,j)}, [\mathbb{R}]_{(k,j)}, M, M\,{}^{(k,j)}\bigcup_{ps} N\}$$

$$\mathcal{T} = \{[\varnothing]_{(k,j)}, [\mathbb{R}]_{(k,j)}, N, M\,{}^{(k,j)}\bigcup_{ps} N\}$$

Then $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$, $\mathcal{T}$ are all (k,j)-picture topologies on $\mathbb{R}$.

Clearly $\mathcal{P}$ is (k,j)-finer than $\mathcal{Q}_{(k,j)}$, $\mathcal{R}_{(k,j)}$, $\mathcal{S}_{(k,j)}$, $\mathcal{T}_{(k,j)}$ each of which is not comparable to other.

5. Conclusion

The extension of union, intersection, and complement of (k, j)-picture sets enhances their theoretical structure and scope. These operations help to reveal the internal behaviour of such sets, though they may differ from classical topological properties. The introduction of topology on (k, j)-picture sets allows the study of continuity and related concepts. Overall, this approach provides a strong basis for further research in generalized set theory.

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Author Declaration

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We further declare that there are **no conflicts of interest** associated with this work. All authors have contributed to the preparation of the manuscript and have approved the final version submitted for publication.