

On Heptadecagon Fuzzy Numbers and their Ranking in Transportation Problem

¹S. Kirupavathi, ²S. Sandhiya*

¹Research Scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Pallavaram, Chennai, Tamil Nadu, India-600117.

Email: kirupavathiseetharaman@gmail.com

²Assistant Professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Pallavaram, Chennai, Tamil Nadu, India-600117.

Email: sandhyasundarr@gmail.com

HOW TO CITE:

S. Kirupavathi, S. Sandhiya (2026).
On Heptadecagon Fuzzy Numbers
and their Ranking in Transportation
Problem.

International Journal of Special
Education, 41(17s), 295-308

COPYRIGHT STATEMENT:

Copyright: © 2026 Authors.
Open access publication under the
terms and conditions
of the Creative Commons Attribution
(CC BY)
license (<http://creativecommons.org/licenses/by/4.0/>).

ABSTRACT:

The Heptadecagon Fuzzy Number (HDGFN), a symmetric 17-point fuzzy structure with exact granularity for uncertainty modelling, is proposed in this study as a novel expansion in the class of polygonal fuzzy numbers. For HDGFN a formal α -cut formulation is established, which allows for the analytical computation of interval-valued representations at different levels of confidence. Furthermore, a innovative ranking function is suggested that is based on weighted symmetric averaging, using this ranking function the fuzzy valued transportation problem is converted to crisp valued problem, which is then solved using the MODI method and a new proposed method. The computational findings indicate that the HDGFN provides a more intricate and expressive modelling framework compared to traditional fuzzy numbers.

Keywords: Heptadecagon Fuzzy Numbers (HDGFN), Fuzzy transportation problem, α -cut of HDGFN.

1. Introduction:

Fuzzy set theory since its establishment by Zadeh in 1965 [28], has emerged as a fundamental framework for modelling imprecise and uncertain information in real-world systems. Fuzzy numbers are pivotal in fuzzy mathematics for describing ambiguous quantities like cost, time, or availability. Conventional fuzzy numbers, such as triangular and trapezoidal fuzzy numbers, are extensively utilized owing to their simplicity and computational efficiency [7]. Nonetheless, their constrained resolution may hinder the modelling of uncertainty when higher fidelity is necessary. To resolve this, polygonal fuzzy numbers have arisen as an extension in which the membership function is formulated using many linear

segments. Polygonal shapes, including pentagonal, heptagonal, and tridecagonal fuzzy numbers, provide a more detailed depiction of uncertainty and are increasingly recognized in fuzzy optimization and decision-making contexts.

Fuzzy transportation problems have been extensively studied because of its significance in representing uncertainty in logistics and supply chain optimization [2, 23]. Researchers have conventionally utilized triangular and trapezoidal fuzzy numbers to denote fuzzy cost, supply, and demand quantities. However, these forms may lack adequate resolution or adaptability in intricate situations. Recent studies have created polygonal fuzzy (PFNs) featuring additional

control points to more effectively represent gradual transitions in fuzzy data [23]. Alrefaei et al. in 2021 introduced a solution approach for the fully fuzzy multi-objective linear programming problems utilizing n -polygonal fuzzy numbers, emphasising their versatility and adaptability in fuzzy modelling [24]. Saxena et al. in 2024 used pentagonal fuzzy numbers in transshipment situations and confirmed their optimality through the MODI approach, demonstrating that polygonal structures surpass simpler fuzzy models. The application of ranking methodologies has undergone substantial evolution [15]. These methodologies underscore the significance of symmetric frameworks and reasoning devoid of defuzzification. Still, the majority of contemporary models continue to depend on established MODI logic modified for discrete contexts.

The MODI approach is a recognized optimization methodology in classical transportation issues, utilized to assess the opportunity costs linked to each unallocated cell in the transportation table. Through the comparison of these opportunity costs, MODI determines if the existing allocation is appropriate or if future modifications can decrease the overall cost [13, 17]. Within the ambiguous framework, we modify the MODI technique by employing ranking values of Heptadecagon fuzzy numbers, thus maintaining the integrity of fuzzy uncertainty during the optimization procedure. This method facilitates a reliable and effective system for attaining near-optimal solutions, even when all problem parameters are ambiguous and depicted by polygonal membership functions [3]. Here we present the heptadecagon fuzzy number (HDGFN), a polygonal fuzzy number characterized by 17 equidistant control points. The HDGFN is especially appropriate for scenarios requiring high-resolution modelling of uncertainty, while ensuring symmetry and computational feasibility. We also establish a closed-form α -cut formulation for the HDGFN, facilitating the analytical derivation of interval bounds for any $\alpha \in [0,1]$. This facilitates fuzzy interval arithmetic and fuzzy decision analysis. Furthermore, we offer a novel ranking algorithm

specifically designed for heptadecagon fuzzy numbers. This function utilizes weighted symmetric averaging of control points, prioritising centre values and exploiting the structure of the HDGFN to derive a precise ranking score without necessitating integration or defuzzification by area. The efficacy of the HDGFN and its ranking model is illustrated in a fuzzy transportation problem, whereby unpredictable costs, supplies, and demands are represented using HDGFNs.

To assess the optimality of the transportation problem generated from fuzzy data, we integrate a fuzzy variant of the MODI approach (Modified Distribution Method). However, when cost coefficients are ambiguous especially represented by intricate framework such as heptadecagon fuzzy numbers conventional MODI methodology is insufficient. We offer a novel optimality verification approach specifically designed for the fuzzy transportation model utilising the ranking values of heptadecagon fuzzy numbers. The findings indicate that the suggested fuzzy number framework facilitates more precise and comprehensible allocation decision compared to traditional fuzzy models.

Research Gap

Even though fuzzy transportation modelling has advanced significantly, current methods mostly rely on computationally demanding ranking techniques and low-order fuzzy numbers [4]. These techniques might not adequately reflect the intricate patterns of uncertainty found in contemporary logistics and transportation systems [5]. Moreover, the use of symmetry preserving ranking methods in conjunction with high-resolution polygonal fuzzy numbers is yet largely studied. This paper presents a Heptadecagon fuzzy number (HDGFN) and a weighted symmetric ranking function (WSRF) for fuzzy transportation issue solving in order to overcome these constraints.

Novelty

The novelty of this work lies in the introduction of a Heptadecagon fuzzy number (HDGFN) with seventeen control points and a new weighted symmetric ranking function (WSRF).

The proposed framework provides a more detailed representation of uncertainty, preserves symmetry during ranking, reduces computational complexity, and offers an effective approach for solving fuzzy transportation problems under uncertain environments.

2. Preliminaries:

2.1. Fuzzy set: [28]

A fuzzy set $\tilde{F}$ in a universe of discourse Y is defined by a membership function:

$$\mu_{\tilde{F}}: Y \rightarrow [0,1]$$

which assigns to each element $y \in Y$ a grade of membership $\mu_{\tilde{F}}(y)$ representing the degree to which y belongs to $\tilde{F}$.

2.2. Fuzzy number: [30]

A fuzzy number $\tilde{F}$ is a fuzzy set on the real number line $\mathbb{R}$ that satisfies:

- $\tilde{F}$ is normal (*i.e.*, $\exists y \in \mathbb{R}: \mu_{\tilde{F}}(y) = 1$)
- $\tilde{F}$ is convex
- $\mu_{\tilde{F}}$ is upper semi-continuous
- The support of $\tilde{F}$ is bounded

2.3. Polygonal fuzzy number (PFN): [25]

A polygonal fuzzy number is a fuzzy number whose membership function consist of piecewise linear segments between multiple control points.

If $\tilde{F} = (a_1, a_2, \dots, a_n)$ is a polygonal fuzzy number with n control points, then the membership function $\mu_{\tilde{F}}(y)$ increase linearly from a_1 to the peak, then decrease linearly to a_n .

Polygonal fuzzy numbers include:

- Triangular fuzzy numbers (3 points)
- Trapezoidal fuzzy numbers (4points)
- Pentagonal, heptagonal and more generally, n -gonal fuzzy numbers.

2.4. α - Cut of a Fuzzy Number:

For a fuzzy number $\tilde{F}$, the α - cut at level $\alpha \in [0,1]$ is the crisp set:

$$\tilde{F}_{\alpha} = \{y \in \mathbb{R} | \mu_{\tilde{F}}(y) \geq \alpha\}$$

If $\tilde{F}$ is convex, then each α - cut is a closed interval:

$$\tilde{F}_{\alpha} = [y_L(\alpha), y_R(\alpha)]$$

2.5. Heptadecagon Fuzzy Number (HDGFN):

A Heptadecagon Fuzzy Number $\tilde{F}^{HDGFN}$ is a fuzzy number characterized by a membership function $\mu_{\tilde{F}^{HDGFN}}(y)$ defined by 17 ordered real numbers:

$$\tilde{F} = (a_1, a_2, \dots, a_{17})$$

Its membership function is defined explicitly as:

$$\mu_{\tilde{F}^{HDGFN}}(y) = \left\{ \begin{array}{ll} 0, & y < a_1 \\ u \left(\frac{y - a_1}{a_2 - a_1} \right), & a_1 \leq y \leq a_2 \\ u, & a_2 \leq y \leq a_3 (\text{plateau}) \\ u + (v - u) \left(\frac{y - a_3}{a_4 - a_3} \right), & a_3 \leq y \leq a_4 \\ v, & a_4 \leq y \leq a_5 (\text{plateau}) \\ v + (w - v) \left(\frac{y - a_5}{a_6 - a_5} \right), & a_5 \leq y \leq a_6 \\ w, & a_6 \leq y \leq a_7 (\text{plateau}) \\ w + (t - w) \left(\frac{y - a_7}{a_8 - a_7} \right), & a_7 \leq y \leq a_8 \\ t + (1 - t) \left(\frac{y - a_8}{a_9 - a_8} \right), & a_8 \leq y \leq a_9 \\ 1, & y = a_9 \\ t + (1 - t) \left(\frac{a_{10} - y}{a_{10} - a_9} \right), & a_9 \leq y \leq a_{10} \\ w + (t - w) \left(\frac{a_{11} - y}{a_{11} - a_{10}} \right), & a_{10} \leq y \leq a_{11} \\ w, & a_{11} \leq y \leq a_{12} (\text{plateau}) \\ v + (w - v) \left(\frac{a_{13} - y}{a_{13} - a_{12}} \right), & a_{12} \leq y \leq a_{13} \\ v, & a_{13} \leq y \leq a_{14} (\text{plateau}) \\ u + (v - u) \left(\frac{a_{15} - y}{a_{15} - a_{14}} \right), & a_{14} \leq y \leq a_{15} \\ u, & a_{15} \leq y \leq a_{16} (\text{plateau}) \\ u \left(\frac{a_{16} - y}{a_{17} - a_{16}} \right), & a_{16} \leq y \leq a_{17} \\ 0, & y > a_{17} \end{array} \right.$$

with parameters clearly satisfying $0 < u < v < w < t < 1$

2.6. α -cut of Heptadecagonal Fuzzy Number (HDGFN):

The α -cut of a heptadecagonal fuzzy number $\tilde{F}^{HDGFN}$ at level α , denoted by $[\tilde{F}^{HDGFN}]_\alpha$, is clearly defined as the crisp set of elements whose degree of membership is greater than or equal to α

$$[\tilde{F}^{HDGFN}]_\alpha = \{y | \mu_{\tilde{F}^{HDGFN}}(y) \geq \alpha, \alpha \in [0, 1]\}$$

Explicitly, the α -cut intervals are given clearly as follows:

$$[\tilde{F}^{HDGFN}]_\alpha = \begin{cases} [HDGFN_{1\alpha}^L, HDGFN_{1\alpha}^U], \alpha \in [0, u] \\ [HDGFN_{2\alpha}^L, HDGFN_{2\alpha}^U], \alpha \in [u, v] \\ [HDGFN_{3\alpha}^L, HDGFN_{3\alpha}^U], \alpha \in [v, w] \\ [HDGFN_{4\alpha}^L, HDGFN_{4\alpha}^U], \alpha \in [w, t] \\ [HDGFN_{5\alpha}^L, HDGFN_{5\alpha}^U], \alpha \in [t, 1] \end{cases}$$

with endpoints defined clearly by:

- For $\alpha \in [0, u]$

$$HDGFN_{1\alpha}^L = a_1 + \frac{\alpha}{u}(a_2 - a_1), HDGFN_{1\alpha}^U = a_{17} - \frac{\alpha}{u}(a_{17} - a_{16})$$
- For $\alpha \in [u, v]$

$$HDGFN_{2\alpha}^L = a_3 + \frac{\alpha - u}{v - u}(a_4 - a_3), HDGFN_{2\alpha}^U = a_{15} - \frac{\alpha - u}{v - u}(a_{15} - a_{14})$$

- For $\alpha \in [v, w]$

$$\begin{aligned} HDGFN_{3\alpha}^L &= a_5 \\ &+ \frac{\alpha - v}{w - v} (a_6 - a_5), HDGFN_{3\alpha}^U \\ &= a_{13} - \frac{\alpha - u}{v - u} (a_{13} - a_{12}) \end{aligned}$$

- For $\alpha \in [w, t]$

$$\begin{aligned} HDGFN_{2\alpha}^L &= a_7 \\ &+ \frac{\alpha - w}{t - w} (a_8 - a_7), HDGFN_{2\alpha}^U \\ &= a_{11} - \frac{\alpha - w}{t - w} (a_{11} - a_{10}) \end{aligned}$$

- For $\alpha \in [t, 1]$

$$\begin{aligned} HDGFN_{2\alpha}^L &= a_8 + \frac{\alpha - t}{1 - t} (a_9 - a_8), HDGFN_{2\alpha}^U \\ &= a_{10} - \frac{\alpha - t}{1 - t} (a_{10} - a_9) \end{aligned}$$

The parameters are clearly set to satisfy $0 < u < v < w < t < 1$, providing flexibility in modelling uncertainty with evidently defined plateau regions.

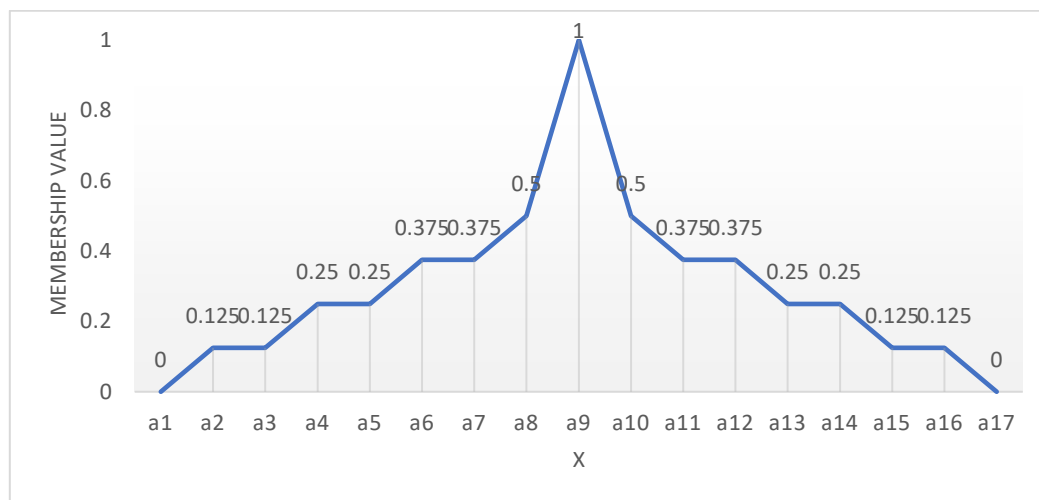
2.7. Weighted symmetric Ranking function (WSRF) (Proposed): [9, 15]

Let $\tilde{F} = (a_1, a_2, \dots, a_{17})$ be a heptadecagon fuzzy number, where a_9 is the peak point, and $\mu(a_9) = 1$, then the weighted symmetric ranking function $R(\tilde{F})$ is defined as:

$$R(\tilde{F}) = \frac{1}{8} \sum_{i=1}^8 \left(\frac{a_i + a_{17-i+1}}{2} \cdot \frac{i}{8} \right) + a_9$$

- $a_i - i^{th}$ point on the left of the fuzzy number
- a_{17-i+1} – symmetric point from the right
- $\frac{i}{8}$ – membership level weight for each symmetric pair
- $\frac{a_i + a_{17-i+1}}{2}$ – average of the symmetric pair
- a_9 – the peak point with membership value 1 (maximum confidence)

Graphical representation of a Heptadecagonal fuzzy number for $u = 0.125, v = 0.25, w = 0.375, t = 0.5$.



3. Theoretical Properties of Heptadecagonal Fuzzy Number and Weighted symmetric Ranking function:

3.1 Validity of Heptadecagonal Fuzzy Number:

Theorem: 1

Let the Heptadecagon Fuzzy Number (HDGFN) $\tilde{H} = (h_1, h_2, \dots, h_{17})$ be defined with ordered control points satisfying $h_1 \leq h_2 \leq \dots \leq h_{17}$ and with a piecewise linear

membership function. Then $\tilde{H}$ satisfies the properties of a fuzzy number, namely:

- Normality
- Convexity
- Upper semi-continuity
- Bounded support

Proof:

- Normality

The membership function of HDGFN attains its maximum value equal to 1 at the central point h_g . Hence,

$$\mu_{\tilde{H}}(h_g) = 1$$

Therefore, the fuzzy set is normal.

(ii) Convexity

The membership function increases monotonically from h_1 to h_9 and decreases monotonically from h_9 to h_{17} .

Thus, for any $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$

$$\begin{aligned} \mu_{\tilde{H}}(\lambda x_1 + (1 - \lambda)x_2) \\ \geq \min \{ \mu_{\tilde{H}}(x_1), \mu_{\tilde{H}}(x_2) \} \end{aligned}$$

Hence, the membership function is convex.

(iii) Upper Semi-Continuity

Since the membership function is piecewise linear over closed intervals, it is continuous on every interval segment. Therefore, it is upper semi-continuous.

(iv) Bounded Support

The support of $\tilde{H}$ is given by

$$Supp(\tilde{H}) = [h_1, h_{17}]$$

Which is bounded because all controls points are finite real numbers.

Hence, HDGFN satisfies all properties of fuzzy number.

Therefore, HDGFN is a valid fuzzy number.

Hence proved.

3.2 Convexity of the α - cut

Theorem 2:

For every $\alpha \in [0,1]$, the α -cut of the Heptadecagon fuzzy number is a closed bounded interval.

Proof

The α - cut of the HDGFN is defined as

$$H_\alpha = \{x \in \mathbb{R} : \mu_{\tilde{H}}(x) \geq \alpha\}$$

Because the membership function is convex and piecewise linear, the set of all points satisfying the above condition forms an interval

$$H_\alpha = [L_\alpha, U_\alpha]$$

Where L_α and U_α denote the lower and upper bounds corresponding to level α .

Since the support is bounded, both bounds are finite.

Therefore, H_α is a closed bounded interval for every $\alpha \in [0,1]$.

Hence proved.

3.3 Properties of the Proposed WSRF

Theorem 3: Monotonicity of WSRF

Let $\tilde{H}_1 = (h_1, h_2, \dots, h_{17})$ and $\tilde{H}_2 = (k_1, k_2, \dots, k_{17})$ be two HDGFNs such that $h_i \leq k_i \forall i = 1, 2, \dots, 17$. Then the weighted symmetric ranking function satisfies

$$R(\tilde{H}_1) \leq R(\tilde{H}_2)$$

Proof:

The weighted symmetric ranking function is defined as

$$R(\tilde{H}) = \sum_{i=1}^8 w_i \left(\frac{h_i + h_{18-i}}{2} \right) + w_9 h_9$$

Where $w_i > 0$ and

$$\sum_{i=1}^9 w_i = 1$$

Since

$$h_i \leq k_i$$

For all symmetric pairs,

$$\frac{h_i + h_{18-i}}{2} \leq \frac{k_i + k_{18-i}}{2}$$

Multiplying by positive weights preserves inequality.

Summing all weighted terms gives

$$R(\tilde{H}_1) \leq R(\tilde{H}_2)$$

Hence, the ranking function is monotonic.

Theorem 4: Linearity of WSRF

For any two HDGFNs $\tilde{H}_1, \tilde{H}_2$ and scalar $\lambda \in \mathbb{R}$,

$$R(\lambda \tilde{H}_1 + \tilde{H}_2) = \lambda R(\tilde{H}_1) + R(\tilde{H}_2)$$

Proof:

Using the definition of WSRF,

$$R(\lambda \tilde{H}_1 + \tilde{H}_2) = \sum_{i=1}^9 w_i (\lambda h_i + k_i)$$

Applying distributive property,

$$R(\lambda \tilde{H}_1 + \tilde{H}_2) = \lambda \sum_{i=1}^9 w_i h_i + \sum_{i=1}^9 w_i k_i$$

Therefore,

$$R(\lambda \tilde{H}_1 + \tilde{H}_2) = \lambda R(\tilde{H}_1) + R(\tilde{H}_2)$$

Hence proved.

Theorem 5: Preservation of Symmetry

If the HDGFN is symmetric about the central point h_9 , then the WSRF reduces to the central value.

Proof:

For a symmetric HDGFN,

$$h_i + h_{18-i} = 2h_9$$

For all symmetric pairs.

Substituting into the ranking function,

$$R(\tilde{H}) = \sum_{i=1}^8 w_i h_9 + w_9 h_9$$

Factoring h_9 ,

$$R(\tilde{H}) = h_9 \sum_{i=1}^9 w_i$$

Since, $\sum_{i=1}^9 w_i = 1$

We obtain, $R(\tilde{H}) = h_9$

Hence proved.

4. Mathematical formulation of a Fuzzy Transportation Problem:

Here we examine the fuzzy transportation problem, which encompasses m sources and n

destinations, with costs, supply, and demand values expressed as fuzzy quantities. Let $\tilde{C}_{ij}$ represent the fuzzy cost of conveying a unit from source S_i to destination D_j , where $\tilde{S}_i$ denotes the source of fuzzy supply accessible at source S_i and $\tilde{D}_j$ signify the fuzzy demand required at destination D_j . All fuzzy values are represented using heptadecagon fuzzy numbers, characterized by 17 control points to ensure a highly detailed and symmetric framework. The decision variable y_{ij} determines the precise quantity to be moved from S_i to D_j .

The aim of the fuzzy transportation challenges is to reduce the overall fuzzy transportation cost. However, as the cost values $\tilde{C}_{ij}$ are concerned, a defuzzification process is necessary for comparison and allocation. To resolve this issue, the suggested model utilized an innovative weighted symmetric ranking function to transform each fuzzy cost into a precise equivalent value, represented as $R(\tilde{C}_{ij})$. The objective function is subsequently articulated as:

$$\min Z = \sum_{i=1}^m \sum_{j=1}^n y_{ij} \cdot R(\tilde{C}_{ij})$$

subject to:

$$\sum_{j=1}^n y_{ij} \approx \tilde{S}_i, \text{ for } i = 1, 2, \dots, m$$

$$\sum_{i=1}^m y_{ij} \approx \tilde{D}_j, \text{ for } j = 1, 2, \dots, n$$

$$y_{ij} \geq 0 \text{ for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

where m is the number of supply points;

n is the number demand points;

$y_{ij} \approx (y_{ij}^1, y_{ij}^2, \dots, y_{ij}^{17})$ is the uncertain number of units shipped from supply point i to demand point j .

$\tilde{C}_{ij} \approx (C_{ij}^1, C_{ij}^2, \dots, C_{ij}^{17})$ is the uncertain cost of shipping one unit from supply point i to demand point j .

$\tilde{S}_i \approx (S_{ij}^1, S_{ij}^2, \dots, S_{ij}^{17})$ is the uncertain supply at supply point i to demand point j .

$\tilde{D}_j \approx (D_{ij}^1, D_{ij}^2, \dots, D_{ij}^{17})$ is the uncertain demand at demand point j .

The transportation model has a feasible solution if:

$$\sum_{i=1}^m \tilde{S}_i \approx \sum_{j=1}^n \tilde{D}_j$$

The fuzzy transportation table can be written as:
Demand Supply

$\tilde{C}_{11}$	$\tilde{C}_{12}$	...		$\tilde{C}_{1n}$	$\tilde{S}_1$
...	...	...	...	...	...
$\tilde{C}_{m1}$	$\tilde{C}_{m2}$	...	...	$\tilde{C}_{mn}$	$\tilde{S}_m$
$\tilde{D}_1$	$\tilde{D}_2$	...		$\tilde{D}_n$	

Table 3.1

5. Procedure for Solving Fuzzy Transportation Problem MODI method:

Step 1: Define the fuzzy transportation problem with m supply sources and n demand destinations and $\tilde{C}_{ij} \approx (C_{ij}^1, C_{ij}^2, \dots, C_{ij}^{17})$ be the heptadecagon fuzzy transportation cost from source i to destination j , $\tilde{S}_i$ and $\tilde{D}_j$ be fuzzy supply and demand values respectively with y_{ij} representing the decision variable.

Step 2: Convert the fuzzy costs into crisp values using the proposed weighted symmetric function ranking formula given by

$$R(\tilde{F}) = \frac{1}{8} \sum_{i=1}^8 \left(\frac{a_i + a_{17-i+1}}{2} \cdot \frac{i}{8} \right) + a_9$$

Step 3: The initial basic feasible solution is found using the least cost method.

Step 4: To apply the MODI method, we first compute the $u - v$ potentials for allocated cells set and we solve

$$u_i + v_j = R(\tilde{C}_{ij})$$

Step 5: For every unallocated cell (i, j) we compute:

$$\Delta_{ij} = R(\tilde{C}_{ij}) - (u_i + v_j)$$

Step 6: The optimality check is performed

If all $\Delta \geq 0$, the solution is optimal, if any $\Delta < 0$, select the most negative Δ

Step 7: The solution of the problem is improved using MODI Loop where a new solution with an improved total cost is obtained. The potentials and opportunity costs are then recomputed, and

the process is repeated until all opportunity cost are non-negative, indicating an optimal solution.

6. Procedure for solving Fuzzy transportation using the Proposed method:

Step 1: Use the ranking function to transform fuzzy values from the provided transportation problem into crisp values.

Step 2: To determine the right top and bottom of the corresponding cost, subtract the minimum cell cost from each cell cost in each row and column of the transportation problem.

Step 3: Add the costs for the right top and right bottom, then enter the total number in the associated cell cost.

Step 4: Find the lowest element in each row and column of the Transportation table, then deduct it from the corresponding row and column.

Step 5: Determine the total of the values in the column and row. Select the maximum penalty amount and place the minimum amount of supply and demand in the row and column's minimum elements. Remove by erasing the rows or columns that represent the locations of supply and demand satisfaction.

Step 6: Keep going through steps 4 and 5 until all supply and demand are satisfied.

Step 7: Align the initial transportation expense with the cell cost that was satisfied.

Step 8: Determine the lowest possible price.

That is Total Cost = $\sum \sum \tilde{C}_{ij} y_{ij}$

7. Formulation of the problem:

In an event of a natural disaster, such as a flood or earthquake, a governmental relief agency is required to allocate emergency supplies from

three regional warehouses to three impacted disaster zones. The transportation costs remain unknown and inaccurate due to persistent road obstructions, fluctuating fuel prices, erratic weather, and real-time supply chain disruptions. The organisation used Heptadecagon fuzzy numbers to more precisely simulate the

transportation costs of relief items between each ware houses and zone, thereby addressing this uncertainty. The objective is to uncertain the ideal transportation strategy that minimizes the total unpredictable costs while satisfying the requisites aid in each zone, utilizing supplies from warehouses.

7.1. MODI method:

Step1: Consider the following fuzzy transportation problem

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)	Supply
W_1	[12,13, ...,28]	$\begin{bmatrix} 10,11,12,13,14, \\ 15,16,17,18,19, \\ 20,21,22,23,24, \\ 25,26 \end{bmatrix}$	$\begin{bmatrix} 14,15,16,17,18, \\ 19,20,21,22,23, \\ 24,25,26,27,28, \\ 29,30 \end{bmatrix}$	$\begin{bmatrix} 100,105.1,110.2,115.3, \\ 125.5,130.6,135.7,140.8 \\ 151,156.1,161.2,166.3, \\ 176.5,181.6 \end{bmatrix}$
W_2	[15,16, ...,31]	$\begin{bmatrix} 13,13.9,14.9,15.8,16 \\ 17.7,18.6,19.6,20.5,2 \\ 22.4,23.3,24.3,25.2,2 \\ 27.1,28 \end{bmatrix}$	$\begin{bmatrix} 11,11.9,12.8,13.6,14.5, \\ 15.4,16.3,17.1,18,18.9, \\ 19.8,20.6,21.5,22.4,23.3 \\ 24.1,25 \end{bmatrix}$	$\begin{bmatrix} 120,125.1,130.2,135.3 \\ 145.5,150.6,155.7,160.8 \\ 171,176.1,181.2,186.3 \\ 196.5,201.6 \end{bmatrix}$
W_3	$\begin{bmatrix} 14,14.9,15.9,16.8,17 \\ 18.7,19.6,20.6,21.5,2 \\ 23.4,24.3,25.3,26.2,2 \\ 28.1,29 \end{bmatrix}$	$\begin{bmatrix} 12,12.9,13.8,14.6,15 \\ 16.4,17.3,18.1,19,19 \\ 20.8,21.6,22.5,23.4,2 \\ 25.1,26 \end{bmatrix}$	$\begin{bmatrix} 16,17,18,19,20, \\ 21,22,23,24,25, \\ 26,27,28,29,30, \\ 31,32 \end{bmatrix}$	$\begin{bmatrix} 90,94.4,98.9,103.4,10 \\ 112.3,116.8,121.3,125.8 \\ 134.7,139.2,143.7,148.2 \\ 157.1,161.6 \end{bmatrix}$
Demand	$\begin{bmatrix} 110,115,120,125,13 \\ 135,140,145,150,15 \\ 160,165,170,175,18 \\ 185,190 \end{bmatrix}$	$\begin{bmatrix} 100,105,110,115,12 \\ 125,130,135,140,14 \\ 150,155,160,165,17 \\ 175,180 \end{bmatrix}$	$\begin{bmatrix} 100,104.7,109.4,114.1, \\ 123.4,128.1,132.8,137.5 \\ 146.9,151.6,156.3,160.9 \\ 170.1,175 \end{bmatrix}$	

Table 6.1

Step 2: Based on the definition of heptadecagon fuzzy number $\tilde{F}$, the ranking function of $\tilde{F}$ is calculated as

The given fuzzy problem is converted into crisp value using the above formula which is given in the table below:

$$R(\tilde{F}) = \frac{1}{8} \sum_{i=1}^8 \left(\frac{a_i + a_{17-i+1}}{2} \cdot \frac{i}{8} \right) + a_9$$

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)	Supply
W_1	31.25	28.125	34.375	220.038
W_2	35.93	32.03	28.125	251.288
W_3	33.59	29.68	37.5	196.601
Demand	234.375	218.75	214.844	

Table 6.2

Step 3: By using the ranked cost matrix and implementing the least cost method, we derive the subsequent initial basic feasible solution:

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)
W_1	1.288	218.75	-
W_2	36.444	-	214.844
W_3	196.601	-	-

Table 6.3

Step 4: Applying the MODI method the u-v potentials are obtained as follows:

u_1	31.25	v_1	0
u_2	35.93	v_2	-3
u_3	33.59	v_3	-7.805

Table 6.4

Step 5: Δ_{ij} values computed for unoccupied cells are given in the table below

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)
W_1	-	-	10.93
W_2	-	-0.9	-
W_3	-	-0.9	11.715

Table 6.5

Step 6: The optimality check is performed using the above-mentioned conditions and the values of the final iterations and total transportation cost is given in the table below:

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)
W_1	220.49	-	-
W_2	13.839	22.60	214.844
W_3	-	196.60	-

Table 6.6

$$\begin{aligned}
 \text{Therefore, the Total Cost} &= (220.49 \times 31.25) + (13.839 \times 35.93) + \\
 &\quad (22.601 \times 32.03) + (214.844 \times 28.125) + (196.60 \times 29.69) \\
 &= 6890.31 + 497.23 + 723.91 + 6042.49 + 5835.09 \\
 &= \text{Rs. } 19,989.03
 \end{aligned}$$

7.2. Proposed method:

Step 1: The fuzzy problem converted to crisp values using the ranking formula is given below

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)	Supply
W_1	31.25	28.125	34.375	220.038

W_2	35.93	32.03	28.125	251.288
W_3	33.59	29.68	37.5	196.601
Demand	234.375	218.75	214.844	

Table 6.7

Step 2: The minimum cost in each row and column, is subtracted from all elements

Row-Minimum Normalized Matrix

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)
W_1	3.125	0	6.25
W_2	7.805	3.905	0
W_3	3.91	0	7.82

Table 6.8

Column-Minimum Normalized Matrix

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)
W_1	0	0	6.25
W_2	4.68	3.905	0
W_3	2.34	1.555	9.375

Table 6.9

Step 3: The normalized matrices are added to form the penalty matrix as given below

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)
W_1	3.125	0	12.5
W_2	12.485	7.81	0
W_3	6.25	1.555	17.195

Table 6.10

After performing the steps 4 and 5 the final allocation table is obtained in step 6

Step 6: The final allocation table is given by

	Z_1 (Urban East)	Z_2 (Hill Region)	Z_3 (Coastal Region)
W_1	220.038	0	0
W_2	14.337	22.107	214.844
W_3	0	196.601	0

Table 6.11

Step 7: The total cost for the transportation problem is given by

$$\begin{aligned} \text{Total Cost} = & (220.038 \times 31.25) + (14.337 \times 35.93) + \\ & (22.107 \times 32.03) + (214.844 \times 28.125) \\ & + (196.60 \times 29.68) \end{aligned}$$

$$= 6876.18 + 515.13 + 708.09 + 6042.49 + 5835.12$$

$$= \text{Rs. } 19,977.0083$$

Step 8: Therefore, the final cost allocation matrix and the total cost is computed.

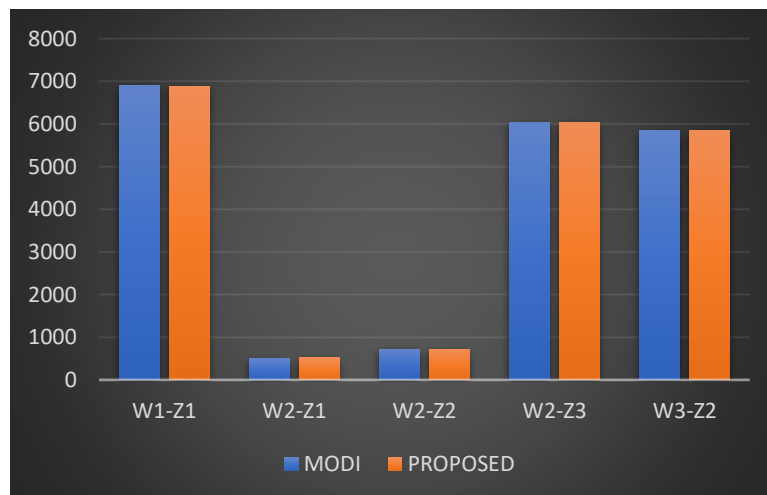
8. Comparison between MODI and Proposed method:

Allocations	MODI method	Proposed method
$W_1 \rightarrow Z_1$	6890.31	6876.1875
$W_2 \rightarrow Z_1$	497.23	515.13
$W_2 \rightarrow Z_2$	723.91	708.09
$W_2 \rightarrow Z_3$	6042.49	6042.49
$W_3 \rightarrow Z_2$	5835.09	5835.12
Total Cost	Rs. 19,989.03	Rs. 19,977.0083

Table 6.12

Results: The final transportation cost table unambiguously shows that the predominant portion of the total expense is derived from three principal allocations

$W_1 \rightarrow Z_1, W_2 \rightarrow Z_3, W_3 \rightarrow Z_2$. The routes contribute and collectively represent almost 95%



Comparison between MODI and Proposed method

Of the total transportation expenditure. Significantly, the complete supply of W_1 is designated to Z_1 , while the totality of W_3 's supply is directed to Z_2 , demonstrating that these were the most economical pathways for their respective supply origins. The allocation technique exhibits optimal cost efficiency by

prioritizing the least expensive routes initially and skilfully balancing remaining resources to reduce total transportation costs.

Conclusion:

This study introduced a novel Heptadecagon Fuzzy Number (HDGFN) framework for modelling uncertainty in fuzzy transportation

problems. The proposed HDGFN, characterized by seventeen control points, provides a higher-resolution representation of uncertain information when compared with conventional polygonal fuzzy numbers. An explicit α -cut formulation was developed to facilitate interval-based analysis and fuzzy computations.

A new Weighted Symmetric Ranking Function (WSRF) was proposed to transform fuzzy transportation parameters into crisp equivalents. The theoretical properties of the proposed ranking function, including monotonicity, linearity, and symmetry preservation, establish its mathematical validity and suitability for optimization applications. The ranking approach avoids complex integration procedures and offers computational simplicity.

The effectiveness of the proposed framework was demonstrated through a fuzzy transportation

problem involving uncertain transportation costs, supplies, and demands. Numerical results indicate that the proposed method produced a lower transportation cost of 19,977.0083 compared with 19,989.03 obtained using the classical MODI approach. This demonstrates the capability of the proposed method to achieve efficient transportation planning under uncertainty.

The proposed HDGFN-WSRF framework provides a flexible and reliable tool for decision-making in logistics, supply chain management, disaster relief operations, and transportation systems. Future research may extend the proposed methodology to multi-objective transportation problems, neutrosophic transportation models, intuitionistic fuzzy environments, network optimization, and Industry 5.0 intelligent logistics systems.

References:

- Arora, A., Gupta, R., & Saxena, R. R., "A technique to solve transshipment problem with asymmetric pentagonal fuzzy numbers" *RAIRO - Operations Research*, vol.58, no.4, pp.3487–3499, 2024. <https://doi.org/10.1051/ro/2024012>.
- Chanas, S., & Kuchta, D., "Fuzzy integer transportation problem", *Fuzzy Sets and Systems*, vol.81, no.3, pp.291–300, 1996. [https://doi.org/10.1016/S0165-0114\(96\)00380-6](https://doi.org/10.1016/S0165-0114(96)00380-6)
- Chatterjee, P., Chakraborty, S., & Pamucar, D., "Recent Advances in Multi-Criteria Decision-Making Techniques," *Operational Research in Engineering Sciences: Theory and Applications*, vol. 8, no. 1, pp. 1–28, 2025.
- Ghatasheh, H., & Alrefaei, M., "An Approach to Solve Fully Fuzzy Multi-Objective Linear Programming Problems With n-Polygonal Fuzzy Numbers", *International Journal of Applied Mathematics*, vol.35, no.6, 2023. <https://doi.org/10.12732/ijam.v35i6.9>
- Gurukumaresan, D., Duraisamy, C., & Srinivasan, R., "Optimal solution of fuzzy transportation problem using octagonal fuzzy numbers", *Computer Systems Science and Engineering*, vol.37, no.3, pp.415–421, 2021. <https://doi.org/10.32604/csse.2021.014130>
- Joshi, V. D., Sharma, M., Čepová, L., & Kalita, K., "Enhance triangular fuzzy parametric framework for solid multi-objective transportation problem with split decision variables," *Scientific Reports*, vol. 15, Article 27474, 2025.
- Kaufmann, A., & Gupta, M. M., "Introduction to fuzzy arithmetic: theory and applications", Van Nostrand Reinhold, 1991.
- Kaur, H., & Kumar, S., "A method for solving fuzzy transportation problem using triangular fuzzy number", *Applied Mathematical Sciences*, vol.7, no.4, pp.175–183, 2013.
- Kaur, A., & Kumar, A., "A new method for solving fuzzy transportation problems using ranking function", *Applied Mathematical Modelling*, vol.35, no.12, pp.5652–5661, 2011. <https://doi.org/10.1016/j.apm.2011.05.012>
- Kumar, R., & Pamucar, D., "A Comprehensive and Systematic Review of Multi-Criteria Decision-Making Methods," *Spectrum of Decision Making and Applications*, vol. 2, no. 1, pp. 177–196, 2025.
- Kumar, S., Singh, A., & Verma, R., "A Novel Ranking Method for Polygonal Fuzzy Numbers and Its Application to Transportation Problems," *Journal of Intelligent & Fuzzy Systems*, vol. 48, no. 2, pp. 1845–1859, 2025.

-
- Kokila, A., & Deepa, G., "Improved fuzzy multi-objective transportation problem with Triangular fuzzy numbers," *Heliyon*, vol. 10, no. 12, e32895, 2024.
- Li, L., Shi, W., Chen, S., Liu, J., Huang, J., & Liu, P., "A Dynamic Adaptive and Resource-Allocated Selection Method Based on TOPSIS and VIKOR," *Neural Processing Letters*, vol. 56, Article 67, 2024.
- Malini, S. U., & Kennedy, F. C., "An approach for solving fuzzy transportation problem using octagonal fuzzy numbers," *Applied Mathematical Sciences*, vol.7, pp.2661–2673, 2013. <https://doi.org/10.12988/ams.2013.13238>
- Mishra, D., Nayak, R. K., & Das, P., "Comparative Analysis of Ranking Functions for Solving Fuzzy Transportation Problems," *International Journal of Fuzzy Systems*, vol. 27, no. 1, pp. 112–126, 2025.
- Mishra, D., & Nayak, R. K., "Solving transportation problem under fuzzy environment using ranking technique," *Journal of Fuzzy Extension & Applications*, vol.1, no.1, pp.35–48, 2020.
- Pamucar, D., Chatterjee, P., & Stević, Ž., "Recent Developments in Fuzzy Multi-Criteria Decision-Making Methods," *Expert Systems with Applications*, vol. 265, Article 125890, 2025.
- Pandian, P., & Natarajan, G., "A new method for solving fuzzy transportation problems," *Applied Mathematical Sciences*, vol.4, no.21, pp.1021–1032, 2010.
- Pramanik, S., Roy, T. K., & Smarandache, F., "Advanced Fuzzy Optimization Techniques for Transportation and Logistics Systems under Uncertainty," *Soft Computing*, vol. 29, no. 7, pp. 5821–5840, 2025.
- Roy, J., & Das, S., "Hybrid Decision-Making Models for Transportation and Resource Planning," *Soft Computing*, vol. 29, no. 4, pp. 3561–3578, 2025.
- Sanjana, R., & Ramesh, G., "A novel approach to interval-valued variables using new interval arithmetic to solve an intuitionistic fuzzy transportation problem," *Journal of Intelligent & Fuzzy Systems*, vol. 43, no. 5, pp. 1–14, 2022.
- Singh, R., "Fuzzy MCDM Models for Logistics and Transportation Planning," *International Journal of Latest Technology in Engineering, Management & Applied Science*, vol. 15, no. 3, pp. 55–66, 2026.
- Taha, H.A., "Operations research an introduction", Pearson Education, 2017.
- Tenguria, A., & Adey, K., "Optimization of Fuzzy Transportation Problems using Triangular Fuzzy Numbers and Ranking Techniques," *International Journal of Mathematics and Computer Research*, vol. 13, no. 10, pp. 5771–5775, 2025.
- Vidhya, V., & Ganesan, K., "A New Ranking Approach for Solving Fuzzy Transportation Problem with Pentagonal Fuzzy Number," *Mathematics and Statistics*, vol. 10, no. 4, pp. 816–824, 2022.
- Wang, Z., & Rangaiah, G. P., "Reference-Based Multi-Criteria Decision-Making Methods: Theory and Applications," *Journal of Decision Systems*, vol. 34, no. 2, pp. 115–132, 2025.
- Yousef, A., Jafar, M. N., & Shaikh, Z. A., "Transportation Problems Using Triangular Type-2 Intuitionistic Fuzzy Numbers," *Engineering, Technology & Applied Science Research*, vol. 16, no. 3, pp. 34907–34913, 2026.
- Zadeh, L. A., "Fuzzy sets", *Information and Control*, vol.8, no.3, pp.338–353, 1965.
- Zhao, Y., Chen, J., & Wang, T., "Assessment of Smart Logistics Systems Using Integrated MCDM Techniques," *Sustainability*, vol. 17, no. 3, Article 1456, 2025.
- Zimmermann, H. J., "Fuzzy programming and linear programming with several objective functions", *Fuzzy Sets and Systems*, vol.1, no.1, pp.45–55, 1978.
-