Between Traditional Schooling and Homeschooling: Criteria for the Legitimation of Mathematical Learning According to Its Actors

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ABSTRACT:

The study examines the criteria through which mathematical learning is recognized and validated in traditional schooling and homeschooling. Unlike research focused on academic performance, it examines the narratives of students, parents/guardians, and teachers alongside mathematical productions from modeling tasks linked to the function object, in order to characterize what counts as evidence of learning, what is valued, who validates it, and through which procedures it is legitimized in each modality.

The study adopts an interpretive qualitative approach through a multiple case study design, involving four triads —two per modality— comprising a student, a parent/guardian, and a teacher; in the homeschooling modality, the mathematics teacher provides support remotely. Data sources included narrative logs, follow-up tasks, an initial diagnostic task, a closing task, students' written productions, and research team observations.

The theoretical framework, articulated synergistically, draws on Legitimation Code Theory, Bernstein's sociology of educational knowledge, Sfard's commognitive theory, and, in a complementary role, Lakoff and Johnson's conceptual metaphor theory, organized around five dimensions: authorized actors, valued knowledge, accepted evidence, validation procedures, and attributed purposes.

Findings show that traditional schooling legitimizes learning through the curriculum, teacher planning, and teacher authority, whereas homeschooling distributes validation among the parent/guardian, the online teacher, digital resources, and external certification. Both modalities share difficulties in legitimizing more complex forms of mathematical learning, such as argumentation and graphical interpretation. The difference

between modalities does not lie in performance, but in the criteria through which learning is validated.

Keywords: homeschooling, traditional schooling, mathematical learning, legitimation of knowledge, validation criteria.

1. Introduction.

In recent years, education systems have undergone transformations that have reshaped how learning is organized in terms of space, time, and form, a diversification that has continued beyond the health emergency (Lee, 2025; Murtha et al., 2025). Factors such as the pandemic, expanding digital technologies, religious and cultural convictions, and experiences of school bullying have driven the search for alternatives to traditional schooling (Batts et al., 2024; Green-Hennessy & Mariotti, 2023; Aliaga, 2017), reconfiguring how families, teachers, and other educational figures participate in children's and young people's education (Papaioannou et al., 2023).

In Chile, learning developed outside the school system can be certified through independent examinations under Exempt Decree No. 2.272, Article 7 (Ministerio de Educacion de Chile, 2007). Internationally, homeschooling is no longer understood solely as a minority practice tied to specific ideological groups but as a more heterogeneous social and educational phenomenon (Cheng & Hamlin, 2023; Bjorklund-Young et al., 2024), although in Chile updated official statistics on its scope remain limited (Centro de Estudios Mineduc, 2023).

Research on traditional schooling and homeschooling has generally compared their effects on academic performance through standardized tests (Valiente et al., 2022; Bosswell, 2021; Tweni et al., 2022), treating learning mainly as a measurable outcome. Findings are not conclusive: some studies report advantages for homeschooling, while Qaqish (2007) found no statistically significant difference, though traditionally schooled students scored slightly higher. Ray's (2017) review of academic performance, social development, and adult success suggests positive effects in some areas but cautions against methodological heterogeneity across studies. These results complicate the assumption that one modality is necessarily better, since learning depends not only on setting but on interactions, instructional time, and opportunities for participation.

Learning mathematics cannot be reduced to obtaining correct answers or a given score: such indicators do not capture how learning is constructed, nor how a student comes to be recognized as someone who knows mathematics within a specific context. Research traditions understand mathematical learning as an active, cumulative, situated process, developed within a community of learners and in relation to a broader sociocultural context (De Corte et al., 1996; Verschaffel et al., 2007). This view aligns with international frameworks, including the Education 2030 Agenda (UNESCO, 2015), ERCE 2019 (UNESCO-OREALC/LLECE, 2021, 2022), and the NCTM (2014), which understand learning mathematics as comprehending, relating, arguing, and validating knowledge, rather than as a scale of results.

This study, therefore, shifts attention away from the exclusive comparison of performance and toward the processes through which mathematical learning is recognized and validated in different educational contexts, a dimension addressed elsewhere through the negotiation of the epistemic norms of a mathematical practice between teachers and students (Lopes et al., 2017), but approached here through Legitimation Code Theory (Maton, 2014, 2016), which allows for a more specific examination of the bases, not always explicit, on which a performance is recognized as achievement.

Building on this, the study seeks to comparatively characterize the criteria through which mathematical learning comes to be recognized as valid in contexts of traditional schooling and homeschooling. The focus is on analyzing how actors narrate, justify, and validate what they

consider to be learning mathematics, while also examining how these judgments relate to students' mathematical productions. This involves attending to which knowledge and procedures are valued, what evidence supports the claim that a student has learned, and who holds the authority to make that judgment.

The study's theoretical framework is organized around the notion of legitimation of knowledge, understood as the process through which an educational practice establishes what counts as valid knowledge, under what conditions it is recognized, and which actors are authorized to declare it (Maton, 2014; Bernstein, 2000).

To account for this complexity, the study articulates three complementary levels: an institutional and structural level, drawing on Legitimation Code Theory and Bernstein's (2000) sociology of educational knowledge; a discursive and interactional level, developed through Sfard's (2008) commognitive theory; and an interpretive level, addressed through Lakoff and Johnson's (1980) conceptual metaphor theory.

To avoid a merely cumulative articulation, these levels are organized following the Foundation, Form, and Function logic (Amador, 2026), which distinguishes the shared epistemological foundations, the form each theory contributes to the analysis, and the comparative function it serves in this study.

From Legitimation Code Theory (LCT), these criteria operate as the "rules of the game" that define what counts as achievement, what knowledge is valued, and which subjects are considered legitimate within a given field (Maton, 2014, 2016).

In particular, LCT's specialization dimension distinguishes between epistemic relations (the relation between a practice and its object of knowledge) and social relations (the relation between the practice and those who take part in it), making it possible to identify different legitimation codes depending on whether specialized knowledge, subject characteristics, both, or neither predominate (respectively, the knowledge code, the knower code, the elite code, and the relativist code). This distinction avoids a simple opposition between traditional schooling and homeschooling, since both codes can combine and shift depending on what the data show.

Basil Bernstein's (2000) sociology of educational knowledge, the framework from which LCT derives, contributes the pedagogic and institutional dimension of legitimation, through pedagogic authority (who may declare that learning has taken place), classification of knowledge (what is properly considered mathematical), and the rules of recognition and realization (how a performance is identified and validated as mathematical).

Commognitive theory (Sfard, 2008) understands mathematical learning as a transformation of discourse: learning mathematics is equivalent to learning to participate in a specialized discourse, characterized by word use, visual mediators, routines, and narratives. Narratives, statements about mathematical objects that are subject to being endorsed or rejected by the community, constitute the unit through which this study accesses the phenomenon.

Lakoff and Johnson's (1980) conceptual metaphor theory, finally, allows for an analysis of the images through which actors organize their understanding of mathematics and learning, based on the projection between a source domain (concrete and embodied) and a more abstract target domain, grounded in image schemas (Johnson, 1987; Lakoff and Núñez, 2000).

Drawing on these contributions, the study operationalizes the criteria for the legitimation of mathematical learning into five analytical dimensions: authorized actors, valued knowledge, accepted evidence, validation procedures, and attributed purposes, whose operational definitions and narrative markers are detailed in the Methodology section.

2. Methodology

2.1 Design

The study followed an interpretive qualitative approach, using a multiple case study design (Stake, 2006), aimed at understanding how criteria for the legitimation of mathematical learning are configured across homeschooling and traditional schooling, without seeking to determine which modality produces better performance, but rather how students, parents/guardians, and teachers recognize, justify, and validate mathematical learning within situated contexts.

2.2 Participants

The unit of analysis was the educational triad, composed of a student, a parent/guardian, and a teacher. Four triads participated, two per modality, with students in the second grade of secondary education, a level at which the function object is part of the national curriculum, allowing comparable content across modalities. Selection was purposive, by convenience, and voluntary, in continuity with a previous phase of the study (Arredondo et al., 2026). Homeschooling students pursued their studies at home with online support in mathematics; traditionally schooled students were enrolled in the subject at a formal institution.

2.3 Instruments

Data production included students' mathematical productions, narrative logs from students, parents/guardians, and teachers, written productions from the sessions, and researcher observations. Mathematical tasks were based on released PISA items (OECD, 2006), adapted to the Chilean curriculum and, in some cases, reformulated from closed to open-ended questions, focusing on modeling phenomena associated with the function object.

2.4 Procedure

The follow-up lasted six weeks. An initial diagnostic task was administered, followed by weekly work sessions and a closing task. These sources allowed contrasting what actors reported about learning with the actual forms of solving, explaining, and validating present in the tasks.

2.5 Data Analysis

The analysis was organized in three stages: a reading by actor, identifying how students, parents/guardians, and teachers narrated mathematical learning and what evidence they considered valid; a within case analysis of each triad, reconstructing how legitimation criteria were articulated in each modality; and a cross case comparison between modalities. Coding combined a deductive orientation from the theoretical framework with an inductive review of emerging nuances, following the data reduction, organization, and comparison logic of Miles, Huberman, and Saldaña (2014).

The cross case comparison was organized around five analytical dimensions, each operationally defined and identifiable through a specific narrative marker:

Authorized actors: who may declare that the student has learned; identified when an actor attributes to themselves or another the authority to issue that judgment.

Valued knowledge: what is recognized as mathematically important (procedures, concepts, understanding, usefulness, argumentation, transfer, or autonomy); identified when actors rank different types of mathematical knowledge.

Accepted evidence: signs considered legitimate indicators of learning, whether externally verifiable or acknowledged by a close third party.

Validation procedures: how evidence of learning is judged (teacher correction, family dialogue, observation, feedback, assessment rubrics, or external certification); identified through the verbs naming the act of evaluating (correct, review, discuss, observe, grade), plus who carries them out and when.

Attributed purposes: the purpose assigned to learning mathematics, whether curricular compliance or personal application.

3. Results

The analysis of results was organized around the five dimensions established in the theoretical framework as criteria for the legitimation of mathematical learning: authorized actors, valued knowledge, accepted evidence, validation procedures, and attributed purposes. These dimensions made it possible to analyze the data produced in the mathematics teaching learning process in traditional schooling and homeschooling, through narrative logs, interviews, teacher lesson plans, a diagnostic task, three interventions, and a closing task based on released PISA items. The sample of participants is described in the Methodology section.

Table 1. Legitimation criteria observed across educational modalities

Dimension	Traditional schooling	Homeschooling
Authorized actors	Teacher (technical narrator role); parents/guardians in a peripheral position, limited to supervising task completion and assessment dates	Distributed network among the parent/guardian (organization and time), the online teacher (weekly session), and external digital resources (tutorials, WhatsApp); specialized mathematical authority remains outside the home
Valued knowledge	Application of formulas and procedures; grades and completion of activities	Application of formulas and procedures, as in traditional schooling; autonomy, time management, and emotional wellbeing are added, valued even above deepening of content
Accepted evidence	Performance on tasks and interventions; brief logs, with little reflection on one's own learning	Performance equivalent to traditional schooling; equally brief logs; greater weight given to attitudinal evidence (motivation, reduced fear and anxiety) as a sign of learning
Validation procedures	Continuous teacher correction; formal summative and formative assessment (1 to 7 scale)	Weekly monitoring by the teacher, follow up by the parent/guardian, occasional support through digital platforms, and, finally, certification through an annual independent examination
Attributed purposes	Passing, complying with the curriculum, maintaining good grades	Building a study environment perceived as safer and better suited to one's own pace

The patterns summarized in Table 1 show that traditional schooling concentrates legitimation criteria in the teacher, the curriculum, and compliance with procedures, while homeschooling combines these criteria with dispositions attributed to the student, such as autonomy, wellbeing, and responsibility.

Authority in validating learning. In traditional schooling, authority rests mainly with the teacher: her lesson planning is organized around the institutional textbook and a mandatory Ministry workbook, and her role shifts between technical and narrator, explaining content and then assigning exercises, with few spaces for dialogue or connection to everyday problems. Parents/guardians AC1 and AC2 exercise, by contrast, a peripheral authority: they supervise task completion and assessment dates ("I tried to keep track of her activities in all subjects and checked whether she had a test or assignment to complete during the week," AC2), but acknowledge that they do not master the current content ("what they are covering at school now, I never learned when I was studying," AC1), and therefore do not intervene in the mathematical validation itself.

In homeschooling, the teacher also operates from a technical narrator role, supported by the same Ministry materials and by weekly video call sessions, with limited room for pedagogical adaptation, although she does incorporate emotional information reported by the parent/guardian to organize the students' time. The parent/guardian AH2, for her part, concentrates her authority in the organizational dimension rather than the epistemic one: "the support I give them at this stage is more about planning and organizational skills," she states, while validation of the mathematical content rests with the online teacher and, ultimately, with the annual independent examination. When doubts arise outside the weekly sessions, the students turn to YouTube tutorials or support through WhatsApp, extending the network of authority to external digital resources beyond the teacher.

Object of legitimation: what counts as mathematical learning. In both modalities, the logs show a marked emphasis on applying formulas and procedures associated with problems addressing the notion of function, drawing on other mathematical concepts present in the problem statements, with little elaboration: "what I found easiest was the rule of three" (EC1); "I found it a bit hard to understand the fractions method" (EH2); "nothing, it was easy for me to understand" (EH1). Only occasionally does a more developed argument appear, as in the case of EC2, who links the volume of a cone and a cylinder to justify how a water tank fills. The modalities differ, however, in the not strictly mathematical forms of knowledge that are also valued: in traditional schooling, grades and completion of activities; in homeschooling, autonomy and time management, together with emotional wellbeing as a condition for learning.

Accepted evidence for recognizing learning. Performance evidence, measured through the problem solving steps applied in the diagnostic task, the interventions, and the closing task, shows no sustained advantage for either modality: in the diagnostic task, the average number of fully completed responses among homeschooling students was two out of four; in the interventions, both modalities averaged two out of five; and in the closing task, both reached three out of five. In both groups, the least achieved step was examining the solution, that is, critically reviewing what had been solved. Narrative evidence confirms this similarity: students in both modalities give brief responses, with little reflection on their own learning. Attitudinal evidence, by contrast, carries more weight in homeschooling: the parent/guardian AH1 states that "the process of studying at home has improved motivation to learn and reduced fear and anxiety toward mathematics," evidence that validates learning without directly referring to a specific mathematical performance.

Validation procedures. In traditional schooling, validation rests on ongoing teacher correction, planning by objectives and content, and the formal assessment system, with a 1 to 7 scale and constant summative and formative evaluations. In homeschooling, weekly monitoring by the teacher, follow up by the parent/guardian, occasional support through digital platforms, and, finally, certification through an annual independent examination are combined, an examination the students themselves experience as a source of nervousness: "the girls, although they answer well, are nervous because next week they have to sit their exams to pass," according to their teacher.

Attributed purposes of learning. In traditional schooling, learning mathematics is associated with meeting a school demand: passing, complying with the curriculum, and maintaining good grades. In homeschooling, by contrast, the initial motivation for choosing this modality is oriented toward building a study environment perceived as safer and better suited to the students' own pace.

4. Discussion

The results of this study support the claim that the difference between traditional schooling and homeschooling is not expressed mainly in better mathematical performance for one modality over the other, but in the criteria through which actors recognize, justify, and validate learning. Performance in the diagnostic task, interventions, and closing task was equivalent between both

modalities, shifting the focus toward understanding how each context organizes recognition of mathematical learning. This finding is consistent with prior research warning against comparing academic performance alone, given the diversity of designs, samples, and evaluation criteria used in this field (Valiente et al., 2022). The contribution of this study lies in showing that mathematical learning can be recognized through different criteria, some tied to specialized knowledge and others to the student's personal and emotional conditions.

From Legitimation Code Theory, this supports the claim that the two modalities differ not only in where learning takes place, but in the bases of legitimation each privileges. Maton argues that educational practices organize rules of the game defining what counts as achievement, what knowledge is valued, and which subjects are recognized as legitimate within a practice (Maton, 2014, 2016). Traditional schooling tends to legitimate mathematical learning from a code close to the knowledge code, centered on procedural mastery and curricular compliance, while homeschooling approaches an elite code, combining specialized knowledge with dispositions attributed to the student, such as autonomy and wellbeing, without fully abandoning procedural criteria. This should be read as a tendency observed in the cases analyzed, not a rule determining in advance how each modality operates.

The authority to validate mathematical learning is distributed differently across modalities. This connects with Bernstein, insofar as the school organizes knowledge through rules of classification and framing that delimit what counts as legitimate content, who transmits it, and under what conditions its learning is recognized (Bernstein, 2000). In homeschooling, authority is reconfigured into a network: the parent organizes time and resources, while specialized validation of mathematical content remains outside the home, with the online teacher and the independent certification exam. This connects directly with the typology of parental engagement proposed by Murphy et al. (2023): parents in both modalities are positioned predominantly as monitors, with occasional facilitation through external resources, and little or no enhancement, that is, active modification of mathematical tasks, showing that modality alone does not determine a more epistemic parental role.

Both modalities also tend to legitimate mathematical learning from predominantly procedural criteria. From Sfard's commognitive perspective, learning mathematics is not only applying procedures but progressively participating in a discourse that involves explaining, justifying, interpreting, and reviewing one's own productions (Sfard, 2008). That examining the solution was, in both groups, the least achieved step, together with the brevity and limited reflection in the logs, suggests this more complex discursive participation remains weakly developed regardless of modality.

Homeschooling, however, expands validation criteria beyond immediate mathematical performance: the parent AH associates it with greater motivation and reduced fear and anxiety toward mathematics, showing that emotional wellbeing can function as evidence of learning on its own. From LCT, this raises a tension: strengthening social relations, the legitimate attributes of the student, risks displacing epistemic relations, the quality of the mathematical knowledge produced (Maton, 2014). This difference also appears in the language each modality uses to name learning: complying and responding in traditional schooling, organizing and sustaining oneself in homeschooling, ways of speaking that, following Lakoff and Johnson, are not neutral (Lakoff & Johnson, 1980).

In practical terms, strengthening mathematical learning in both modalities requires going beyond correcting answers or completing tasks, explicitly incorporating argumentation and critical review of procedures as recognized validation criteria. For education policy, official recognition of homeschooling learning, currently centered on independent examination, could benefit from guidance making the required epistemic criteria

explicit, without displacing the autonomy and wellbeing families value.

Among the limitations of this study, the sample was small: three traditional schooling students with complete data, after the withdrawal of a fourth participant, and two homeschooling students who are sisters, with a single teacher and one or two parents per modality, which prevents generalizing beyond the cases studied. Purposive, convenience-based selection and the qualitative, interpretive design reinforce the need to read these results as situated tendencies rather than consolidated regimes of legitimation. Finally, the data were originally produced for another purpose and reread here through a different framework, which limits the depth of some interpretations and points to studies designed explicitly around the five dimensions proposed here.

5. Conclusion

This study allows us to conclude that the criteria for legitimating mathematical learning are configured differently in traditional schooling and homeschooling, without this translating into better mathematical performance for one modality over the other. In traditional schooling, authority and validation are concentrated in the teacher, the curriculum, and the formal assessment system; in homeschooling, they are redistributed across a network in which the parent organizes time and resources, the online teacher resolves specific questions, and final certification rests on an independent examination outside the home. Both modalities, however, share a substantive limitation: they legitimate mathematical learning mainly through procedural criteria, with limited development of argumentation and critical review of solutions.

The contribution of this study does not lie in establishing which modality is more effective, but in showing that mathematical learning can be recognized through different criteria: some more closely tied to specialized knowledge, and others to the student's organizational and emotional conditions, such as autonomy or wellbeing, which in homeschooling come to function as evidence of learning in their own right. This shift is relevant for mathematics education, as it questions what is being validated when a student is said to have learned mathematics, and shows that expanding the spaces in which learning takes place is not enough unless the criteria through which that learning is recognized as mathematically meaningful are also reconsidered.

Given the small size of the sample, these findings should be understood as an initial exploratory approach rather than as generalizable conclusions. Future studies could be designed explicitly around the five dimensions proposed here, with larger samples in both modalities, incorporate audiovisual records of the interaction among students, parents, and teachers, apply tasks with greater cognitive demand, and further examine how students transition from procedural forms of problem solving toward more argumentative and interpretive mathematical practices. It would likewise be valuable to explore how guidance that makes the epistemic criteria of homeschooling certification explicit could strengthen mathematical learning without displacing the autonomy and wellbeing that families value in this modality.

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