Rebuilding Number Sense: Teaching Euclid's Algorithm and LCM through Guided Practice in Prospective lower secondary mathematics teachers

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ABSTRACT:
This study analyses the impact of two guided-practice-based teaching strategies rediscovering the Euclidean Algorithm and Adding Fractions with Meaning on the conceptual understanding and development of didactic-mathematical competencies in 30 future mathematics teachers in Panama. The intervention was grounded in the Didactic-Mathematical Knowledge and Competencies model (DMKC) and implemented over five sessions within a university-level Mathematics Didactics course linked to the broader formative framework of the EMAS program. The study aimed to address the gap between procedural repetition and structural understanding, particularly in topics such as greatest common divisor (GCD) and least common multiple (LCM), which are central to the teaching of fractions and divisibility, as well as to the development of numerical reasoning in lower secondary mathematics. Initial results showed that over 70% of the participants were unable to justify concepts such as multiples or common divisors, revealing relevant conceptual weaknesses. Throughout the intervention, participants used pseudocode and solved contextualized problems. By the end of the third session, 66% had constructed a functional pseudocode for the Euclidean Algorithm, and in the post-test, 73% correctly solved fraction problems using the LCM, with 64% clearly justifying their procedures. Qualitative findings indicate strong pedagogical appropriation: 93% of participants proposed meaningful didactic applications for lower secondary mathematics classrooms. The data suggest progress in multiple dimensions, particularly in supporting the integration of computational thinking and historical algorithms as formative resources in mathematics prospective mathematics teachers. This intervention forms part of a broader doctoral research project aimed at strengthening mathematics teacher education in Panama. Its findings contribute to the redesign of formative cycles within EMAS and to the development of a blended specialization in mathematics didactics that integrates inclusive principles (CID-DUA), computational thinking, and frameworks such as TPACK-XK and the Theory of Instrumental Genesis.

Keywords: Euclidean algorithm, least common multiple, mathematics teacher preparation, lower secondary mathematics education, guided practice, pseudocode, conceptual understanding, DMKC model.

1. Introduction

The teaching of foundational mathematical concepts has often been dominated by approaches centred on the mechanical repetition of procedures, leaving conceptual understanding and structural exploration in the background. This tendency continues to affect school mathematics learning and poses significant challenges for prospectives mathematics teachers in Panama particularly in relation to the teaching of divisibility, fractions, and numerical reassign in lower secondary education. According to the PISA 2022 report, more than 74% of Panamanian students scored below level 2 in mathematical competency, indicating serious difficulties in solving tasks that require numerical reasoning and understanding of basic structures (OECD, 2023). Romberg (as cited in Socas & Camacho, 2003) pointed out that Mathematics has been conceived in multiple ways: as a symbolic language, a logical system, a calculation technique, a structure of thought, or a scientific model. In contemporary mathematics education, it is necessary to to move beyond the unreflective memorization of algorithms and to emphasize mental processes driven by reasoning, creativity, and curiosity (Socas & Camacho, 2003; Segarra, 1999). From this perspective, mathematics education requires didactic strategies that connect formal structures with meaningful situations. Nevertheless, for many learners, mathematics continues to be perceived as a difficult subject, largely because of the abstraction od its language and the predominance of decontextualized content (López Rodríguez, 2002).

One of the content areas most affected by this mechanistic approach is the treatment of greatest common divisor (GCD) and least common multiple (LCM), both of which are central to the teaching of fractions, proportions, divisibility, and numerical reasoning. In many schools' contexts, these concepts are introducing as decontextualized formulas, disconnected from their logical structure, applicability, and historical-pedagogical value (Fauzan et al., 2020). This disconnection limits the development of number sense and algorithm thinking. Several studies have therefore argued for rethinking the teaching od these topics trough strategies that promote active learning, computational thinking, and the use of tools such as pseudocode to represent mathematical procedures. In particular, Sabrigiriraj (2024) argues that the Euclidean Algorithm can be reappropriated from a computational perspective, allowing learners to understand its internal logic and transfer it to new contexts. Similarly, Li and Tsai (2022) have shown that problem-based learning supports stronger performance in topics such as GCD and LCM.

This study is based on the implementation of two guided-practice instructional activities: Rediscovering the Euclidean Algorithm and Adding Fractions with Meaning. These activities were designed within the framework of the Didactic-Mathematical Knowledge and Competencies model (DMKC), which integrates epistemic, cognitive, ecological, interactional, mediational, and emotional knowledge in order to promote critical, reflective, and context-sensitive mathematics teaching (Godino, 2013; Pino-Fan et al., 2018; Pino-Fan et al., 2023).

Within this perspective, recent proposals have emphasized the importance of integrating inclusive principles with tools for didactic reflection in mathematics education. Specifically, the articulation between Universal Design for Learning (UDL) and Didactic Suitability Criteria (CID) offers a framework that allows educators to attend to student diversity without compromising the epistemological and cognitive quality of mathematical instruction (Sánchez, Ledezma, & Font, 2025). This integrative perspective informs the present study, which, seeks not only to enhance conceptual understanding, but also to promote reflective, inclusive, and theorically grounded teaching practices.

The intervention was developed in articulation with the EMAS program (Mathematics Education Applied to Secondary Education) and involved 30 future mathematic teachers from a public university in Panamá enrolled in Mathematics Didactics course. Across five class sessions, participants progressively constructed meaning around GCD and LCM through contextualized problem-solving, the use of multiple representations, and the creation of pseudocode as a way to systematize mathematical reasoning. The research question guiding this study is the following: To what extent does the integration od the Euclidean Algorithm and the calculation of the least common multiple, through guided practice, enhance conceptual understanding and the development of didactic-mathematical competencies in future mathematics teachers preparing for lower secondary teachen contexts? This question is consistent with the broader need to strengthen mathematics prospective mathematic teachers for secondary education in Panama.

Furthermore, this work aligns with educational approaches that promote the integration of discipleinary knowledge, culture, and pedagogical practice. As Panou and Violetis (2018) argue, the use of historical or cultural structures to teach abstract concepts makes it possible to resignify knowledge and connect it to learners' experiences. In this sense, the present study responds to the didactic and social need for mathematics teaching that is meaningful, purposeful, and educationally relevant. Its contribution lies in combining the guided rediscovery of a historical algorithm with the structured development of computational thinking through pseudocode in mathematics teacher preparation oriented toward lower secondary teaching. Unlike prior works that isolate either content knowledge or technological tools, this intervention proposes a theory-driven model that operationalizes didactic-mathematical competencies through active and reflective classroom practice.

From an educational perspective, understanding structures such as the greatest common divisor and the least common multiple constitutes an important step toward the development of functional thinking. Both concepts involve recognizing patterns, dependency relations, and invariant structures that may later be formalized through more advanced mathematical representations. In this regard, engaging with algorithms and pseudocode does not merely strengthen procedural learning; it also contributes to the construction of mathematical meaning and supports the transition from arithmetic reasoning to more abstract forms of thought. This progression is especially relevant in mathematics teacher preparation, where a structurally grounded understanding of mathematical objects supports the quality of future teaching practices and the interpretation of mathematical phenomena across school contexts.

2. Theoretical Framework

The model of Didactic-Mathematical Knowledge and Competencies (DMKC) serves as the core framework of this study. Drawing on didactic-mathematical knowledge and competency frameworks developed in mathematics teacher education research (Pino-Fan et al., 2018; Pino-Fan et al., 2023), the DMKC model encompasses a structure composed of six interconnected types of knowledge: epistemic, cognitive, ecological, interactional, mediational, and emotional. These domains interrelate in practice through professional competencies that enable teachers to critically and contextually analyze, plan, implement, and assess mathematics teaching and learning processes. Epistemic knowledge refers to a deep understanding of mathematical objects, including their properties, meanings, and interrelationships. In the case of stude (GCD) and least common multiple (LCM), this knowledge is reflected in grasping their logical structure, historical origins, and relevance in solving real-world problems. Cognitive knowledge, in turn, involves the ability to anticipate and analyze the mental processes involved in student learning—crucial for identifying common errors in topics such as divisibility and fractions (Godino, 2013).

Didactic knowledge refers to the ability to transform mathematical content into accessible and meaningful teaching situations. In this study, the selection of contextualized problems and the use of the Euclidean Algorithm as a discovery tool demonstrate a deliberate effort to reframe the learning of foundational concepts. Evaluative knowledge was activated through rubrics that assessed not only performance outcomes but also the quality of participants' justifications. Likewise, technological knowledge was evident in the integration of pseudocode as a structured representation of the Euclidean Algorithm, facilitating alignment with computational thinking. This dimension becomes increasingly relevant in an educational context where digital competencies are essential for meaningful learning mediation (Nordby et al., 2022; Rodríguez-Martínez et al., 2020). Finally, curricular knowledge allowed the proposed activities to be aligned with the learning objectives of the Panamanian curriculum, particularly regarding fractions, multiples, and divisibility.

The DMKC framework is further enriched by the Onto-semiotic Approach to Mathematical Knowledge and Instruction (EOS), which provides tools to analyze the meanings constructed within classroom practices. According to this perspective, meaning can be understood in terms of systems of operational and discursive practices, made operative through epistemic and cognitive configurations situated within institutional and cultural contexts (Godino, 2013). Accordingly, the activities designed in this study were not merely aimed at teaching procedures but at generating new configurations of meaning relevant to future teaching practice. Recent literature reinforces the necessity of integrating computational and historical perspectives into mathematics teacher education. For instance, Nordby et al. (2022) and Rodríguez-Martínez et al. (2020) highlight how computational thinking and pseudocode development contribute to structural mathematical understanding and pedagogical transfer. Likewise, Pino-Fan et al. (2023) emphasize the importance of didactic-mathematical competence models in guiding teacher professionalization. These findings align with the present study's use of the DMKC framework as both an interpretative and design tool, rooted in contemporary international educational research.

3. Methodology

This study follows a qualitative interpretative approach, incorporating elements of educational action research and focuses on the preparation of the prospective mathematic teachers for lower secondary education. the sample consisted of 30 participants (22 woman and 8 men) from a public university in Panamá, who engaged in sequence of five guided-practice sessions during the first semester od the mathematic didactics Couse NCMA 0011. This instructional strategy integrated activities focused on the concepts of greatest common divisor (GCD) and least common multiple (LCM), designed to promote the development of didactic-mathematical Competencies through contextualized problem-solving and situated pedagical reflection. The instructional design also drew upon extended frameworks such as TPACK-XK and the Theory of Instrumental Genesis (TIG) ro ensure coherence between technological mediation, content knowledge, and pedagogical strategies in the guided practice sequences.

We qualitatively analyzed students' productions and feedback processes. The design aimed to approximate real-world teacher training scenarios, fostering formative assessment that reveals transformations in conceptual understanding, algorithmic thinking, and affective disposition toward teaching fundamental mathematical content. Two complementary didactic sequences were implemented: (1) Rediscovering the Euclidean Algorithm, focused on calculating the GCD from a logical-constructive perspective; and (2) Adding Fractions with Meaning, centered on the use of the LCM to add heterogeneous fractions from a computational and structural perspective. Both sequences were implemented across five consecutive 40-minute sessions.

The instructional strategy was grounded in guided practice, integrating problem-based learning, visual scaffolding (e.g., number lines, fraction bars, and step tables), collaborative work, and the progressive construction of pseudocode (Guía para el Diseño de un Artículo Científico, 2024). Rather than presenting definitions and formulas upfront, the activities were designed to elicit intuitive reasoning through real-life scenarios that required students to explore number relationships and reconstruct procedures logically. For example, students were tasked with solving contextual problems such as synchronizing runners or dividing ribbons without leftovers, which led them to discover the need for common multiples or divisors. Gradually, they formalized these intuitive strategies into structured algorithms, culminating in the co-construction of pseudocode representations.

This study forms part of a broader doctoral research project on mathematics teacher education in Panama and provides empirical evidence on the development of didactic-mathematical knowledge and competencies through guided practice, pseudocode construction, and reflective instructional design. This iterative and reflective approach fostered not only procedural fluency but also a deeper conceptual grasp of divisibility, allowing prospectives teachers to bridge mathematical theory with pedagogical application.

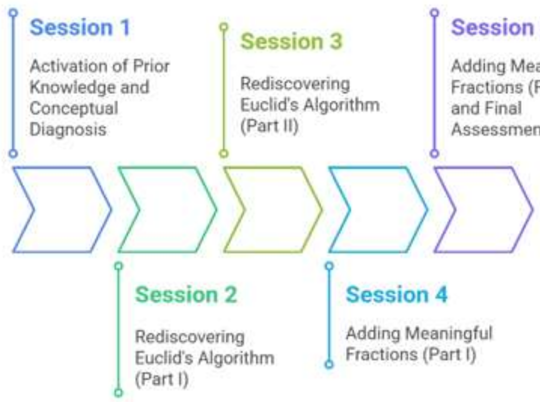

Figure 1. Instructional design of the activities "Rediscovering the Euclidean Algorithm" and "Adding Fractions with Meaning

Data collection involved individual worksheets, self-assessment guides, the instructor-researcher's field notes, and group observation matrices. Across the five sessions, we documented written student work, pseudocode representations, and final reflections, which were organized using a coding system based on the DMKC model. Student responses were assessed using a custom rubric that evaluated six key dimensions.

Data were systematized through double-entry matrices and comparative analysis by student and group. No specialized statistical software was used, given the interpretative approach, but descriptive statistics were processed using basic tools (Excel and Python). This triangulation of data sources enhances internal validity and supports the replicability of the experience.

The custom rubric included six indicators—understanding of GCD, LCM, fractions, clarity of explanation, pedagogical application, and pseudocode construction—each rated on a three-level scale (low, medium, high). The rubric was pilot-tested for clarity and coherence, and applied consistently across sessions. Appendix 1 provides sample tasks and anonymized student responses per indicator, allowing transparency and replicability.

Session 1 – Activation of Prior Knowledge and Conceptual Diagnosis: The initial session aimed to identify participants' prior knowledge through contextualized problems on divisibility. An individual pretest and a group discussion were conducted to explore notions such as multiples, divisors, and number relationships. Results showed that 57% of the participants relied on mechanical strategies without clear conceptual understanding of GCD and LCM, aligning with low performance levels reported in PISA 2022 for Panama (OECD, 2023).

Figure 2. Initial diagnostic assessment: Exploring prior knowledge on divisibility

Session 2 – Activity 1: Rediscovering the Euclidean Algorithm (Part I): This session introduced the problem: "What length should the cuts be to divide two ribbons of 30 cm and 12 cm without leftover material?" Without mentioning the algorithm, students worked in groups using manipulatives, graph paper, and tables for repeated subtractions. The goal was to induce logical reasoning through number patterns. By the end, the groups verbally reconstructed the Euclidean Algorithm by subtraction and collaboratively wrote an initial pseudocode.

In a subsequent session, students were presented with a contextualized problem: "Two students run around a circular track. One takes 24 minutes to complete a lap, the other 36 minutes. When will they meet again at the starting point?" They were asked to determine the answer without referencing GCD or LCM. Participants initially used repeated addition and multiplication, then moved on to constructing multiplication tables and using repeated subtraction.

Session 3 – Activity 1: Rediscovering the Euclidean Algorithm (Part II): This session focused on the division-based version of the Euclidean Algorithm. Students worked in pairs on new problems (e.g., 84 and 36; 121 and 11) and created their own pseudocode either on paper or on a computer. A custom rubric assessed precision, logical clarity, and applicability. Sixty-six percent of groups scored at medium or high-performance levels. As students noticed patterns in their calculations, they realized that these related to the LCM. Subsequently, the formula: $lcm(a,b) = \frac{a \cdot b}{gcd(a,b)}$ was introduced without explaining how to find the GCD. Students were challenged to find the GCD of 24 and 36 via repeated subtractions, leading to an autonomous reconstruction of the Euclidean Algorithm by difference (Sabrigiriraj, 2024) and later by division. They completed sequential tables using the division algorithm to find the GCD, then applied the LCM formula to solve the original problem.

Session 4 – Activity 2: Adding Fractions with Meaning (Part I): Building on their understanding of the GCD, students were introduced to the LCM as a bridge toward adding fractions. Exercises with heterogeneous fractions were solved, emphasizing the use of LCM obtained through the formula. The session incorporated guided practice and visual tools such as fraction strips and number lines. Students linked arithmetic operations with logical structures previously learned.

Session 5 – Activity 2: Adding Fractions with Meaning (Part II) and Final Evaluation: A series of integrative problems were presented requiring students to apply the GCD to find the LCM, justify procedures, and perform fraction additions. Participants wrote pseudocode to represent the complete process and reflected on the pedagogical applications of their learning. A posttest was administered, along with a self-assessment guide focusing on perceived learning.

Data were analyzed qualitatively using categories from the DMKC model (epistemic, cognitive, affective), supported by a comparison of pretest and posttest scores to observe individual and group progress. The evaluation process involved three main instruments: a diagnostic pretest on divisibility, a structured worksheet with six sequential exercises (including pseudocode construction), and a reflective self-assessment guide. The structure and content of the self-assessment guide were inspired by current validated frameworks that support reflective teaching. In particular, the CID–DUA guide served as a reference for integrating inclusive and didactic criteria in the design of reflection tools. This guide was developed through document analysis, expert triangulation with high agreement, and validation by researchers in mathematics education, which reinforces its methodological robustness and relevance in formative contexts (Sánchez, Ledezma, & Font, 2025). The rubric evaluated six indicators (GCD, LCM, fractions, explanation, application, algorithm), each with three performance levels (high, medium, low). Appendix 1 includes a table with sample items and real student responses categorized by performance level. This documentation seeks to enhance methodological transparency and provide a reference for future interventions in initial teacher education contexts.

To ensure methodological robustness, data triangulation was achieved by cross-referencing three main data sources: pre- and post-assessment instruments, classroom artifacts (such as pseudocode constructions and self-assessments), and instructor field notes. This approach enabled the validation of emerging patterns in conceptual development, and mitigated reliance on any single source. The interpretative analysis was further enhanced by peer debriefing among the research team, strengthening the internal consistency of findings.

Given that the main instructor was also part of the research team, several measures were taken to reduce potential biases. These included the anonymization of student work before evaluation, double coding of qualitative data by independent researchers, and reflective journaling to document the researcher's assumptions and decisions throughout the study. These strategies aimed to safeguard the integrity and objectivity of the data analysis process.

4. Results

The outcomes obtained over the five instructional sessions reveal significant, though heterogeneous, progress in the conceptual understanding of the greatest common divisor (GCD) and least common multiple (LCM), as well as in the prospectives teachers' ability to integrate these concepts into classroom practice. In the initial pretest, 70% of participants were unable to adequately justify the difference between a common divisor and the greatest common divisor. Only 13% correctly applied the GCD to a contextualized problem.

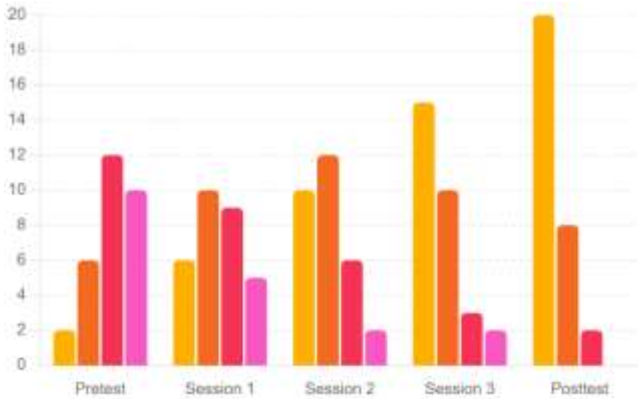

Figure 3. Performance distribution of 30 prospectives teachers across five instructional sessions on GCD and LCM.
(Source: Authors' elaboration based on study activity results)

By the end of the third session, 66% of participants had successfully constructed a functional pseudocode for the Euclidean Algorithm. However, only 43% managed to do so without direct assistance. These findings indicate that guided practice supported by visual scaffolding (e.g., tables, number lines, and fraction blocks) facilitates the construction of enduring mathematical meaning, although systematic validation of the procedure still needs reinforcement. This result aligns with studies emphasizing the importance of scaffolding in contexts where mathematical literacy is limited (Li & Tsai, 2022; Suarsana, 2021).

In the final post-test, 73% of participants correctly solved problems involving the addition of unlike fractions using the least common multiple (LCM) calculated via the Euclidean Algorithm, demonstrating a solid grasp of the structural connections between divisibility and fraction operations. Moreover, 64% of the participants provided clear, well-reasoned justifications for their procedures, reflecting improvements not only in computational accuracy but also in mathematical communication and explanatory skills. These results suggest that the guided practice model effectively bridged conceptual understanding with algorithmic execution. In terms of total rubric scores (with a maximum of 16 points), the average score increased markedly from 6.3 in the pretest to 12.1 in the posttest, indicating substantial gains in overall performance across multiple dimensions of didactic-mathematical competence. This growth reinforces the value of integrating historical algorithms and pseudocode construction into teacher preparation to enhance both mathematical content knowledge and pedagogical proficiency.

These advances were evident not only in the assessment scores but also in the quality of student productions. For instance, analysis of final challenge tasks revealed that 83% of participants were able to apply the Euclidean Algorithm correctly without direct instruction, designing pseudocode that demonstrated structured reasoning. This finding underscores the value of guided practice as a formative learning environment, enabling prospectives teachers to move from mechanical reproduction toward a meaningful reconstruction of fundamental mathematical procedures. Qualitative analysis showed that participants who used visual representations (such as tables, diagrams, or natural language) to illustrate the algorithm's steps achieved higher performance levels, highlighting the importance of multiple representations in developing algorithmic thinking.

Furthermore, written reflections from participants provided strong evidence of pedagogical appropriation of the content. A total of 93% of prospectives teachers referenced potential applications of GCD and LCM in lower secondary mathematics classrooms, including examples related to numerical reasoning, proportional situations, classroom scheduling, and the organization os contextualized tasks involving divisibility and fractions. These responses reveal not only conceptual understanding but also a favorable affective and instructional disposition toward teaching these concepts. Notably, participants with stronger performance were also more likely to contextualize their learning and propose real-world instructional scenarios. This confirms that the integration of computational thinking into teacher education, when paired with reflective scaffolding, can enhance the development of critical professional competencies for contemporary secondary education.

Table I. Performance of 30 prospectives Teachers During Instructional Sessions

Session	High (≥14 pts)	Medium (10–13 pts)	Low (6–9 pts)	Insufficient (<6 pts)
Pretest	1	3	5	21
Session 1	2	6	12	10
Session 2	4	10	10	6
Session 3	6	14	8	2
Session 4	8	16	5	1
Posttest	13	11	5	1

Figure 4 illustrates the comparative performance of 30 prospectives teachers across two instructional strategies: Rediscovering Euclid and Adding Fractions with Meaning. The radar chart displays the distribution of participants by performance level—High (≥14 pts), Medium (10–13 pts), Low (6–9 pts), and Insufficient (<6 pts)—based on the final evaluation rubric applied to both activities.
The results reveal a notably higher concentration of high-performing participants in the "Adding Fractions with Meaning" group, where 18 out of 30 students achieved the highest category, compared to only 10 in the "Rediscovering Euclid" group. Conversely, the Euclid-based activity showed a more balanced distribution across Medium and Low performance levels, with a slightly higher proportion of participants falling into the Insufficient category.
This comparison suggests that the second instructional strategy—more directly tied to fraction operations and classroom applicability—enabled a greater number of students to consolidate their learning and demonstrate mastery. Nonetheless, the Euclid activity facilitated stronger engagement with algorithmic thinking, particularly for those with intermediate conceptual understanding. These patterns reinforce the complementary nature of both strategies in developing didactic-mathematical competencies.

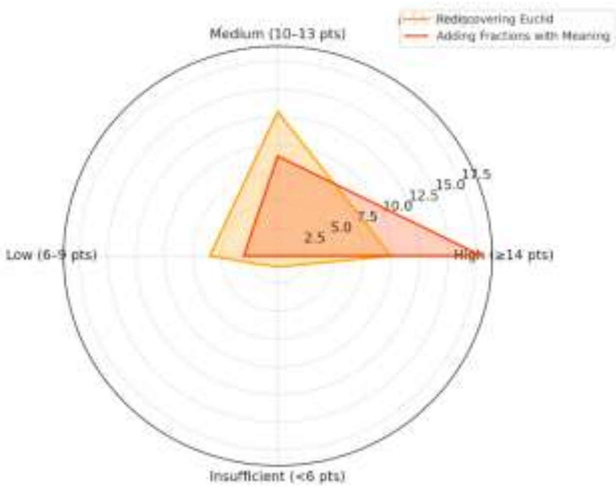

Figure 4. Performance Comparison by Instructional Strategy
(Source: Authors' elaboration based on study activity results)

In the qualitative analysis of participants' written reflections, notable improvements were identified across three key dimensions: epistemic (recognition of the GCD as a logical structure), cognitive (use of diverse and well-reasoned problem-solving strategies), and affective (expressions of satisfaction and increased confidence). A total of 85% of the prospectives teachers indicated that they would teach these mathematical concepts differently following the experience.

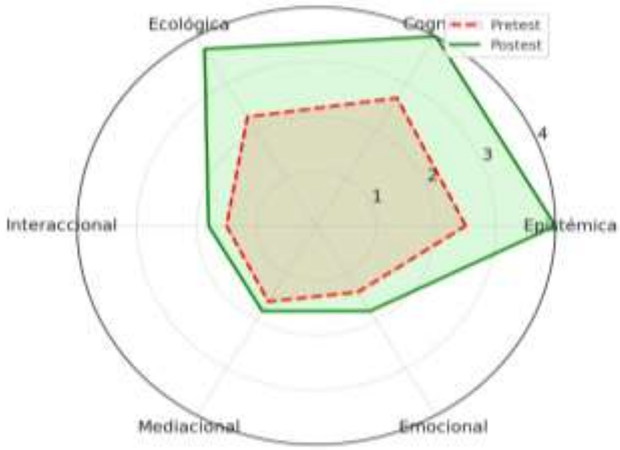

Figure 5. Average performance by DMKC model dimension among prospectives teachers
(Source: Authors' elaboration based on study activity results)

Figure 5 presents a radar chart comparing the average performance across the six dimensions of the DMKC model before and after the instructional intervention. A clear improvement was observed in all dimensions, with the most pronounced gains in the emotional, interactional, and mediational dimensions—those that initially recorded the lowest levels. The epistemic and cognitive dimensions remained high in both the pretest and posttest, suggesting a relatively stronger initial mastery of conceptual and procedural mathematical knowledge. However, the growth observed in the dimensions associated with interaction, pedagogical mediation, and emotional security indicates that the guided practice fostered a comprehensive development of didactic-mathematical competencies.

Despite these gains, some persistent difficulties were noted, particularly in the automatization of the division-based version of the Euclidean Algorithm and in accurately translating mathematical reasoning into pseudocode for more complex problems. These findings suggest a need for additional sessions of guided practice to reinforce computational fluency and algorithmic thinking in teacher preparation context oriented to lower secondary education.

Despite overall gains, two persistent difficulties emerged: (1) translating division-based reasoning into accurate pseudocode, and (2) sustaining abstraction across novel problem contexts. Some students defaulted to mechanical processes, indicating partial internalization. These challenges point to the need for extended scaffolding and differentiated instruction, especially in bridging procedural fluency with algorithmic generalization. The insights generated from this phase will inform the redesign of subsequent implementation of new formative cycles within the broader doctoral research project and their articulation with EMAS training framework

The results of this study underscore early indications of developmental progress in didactic–mathematical knowledge among participants, particularly concerning their ability to connect algorithmic reasoning, symbolic representation, and conceptual justification. Although a specific analysis of functions is not undertaken, observed activities—such as the incremental reconstruction of procedures, the deliberate use of pseudocode, and the establishment of links among quantitative relationships—constitute fundamental groundwork for future development of functional thinking. These findings provide empirical support for the design of more advanced formative experiences, wherein the analysis of functions can be constructed upon a robust and conceptually meaningful foundation.

5. Discussion

The results of this intervention align with studies that have demonstrated how discovery-based learning and programming approaches enhance mathematical understanding at both school and university levels (Fauzan et al., 2020; Sabrigiriraj, 2024). They also reinforce the findings of Nordby et al. (2022) regarding the potential of computational thinking to strengthen structural mathematical skills.

Compared with prior research conducted in school settings on fractions, divisibility, and, computational thinking, this study shows that the use of algorithms and pseudocode is also feasible and pedagogically productive in mathematics teacher preparation. The key distinction lies in the reflective rather than merely technological, orientation of this study, and in the integration of the DMKC framwork as a structural support for mathematical understanding and teaching.

Moreover, this proposal is supported by the findings of Valbuena-Duarte et al. (2021), who highlight that the development of logical-mathematical intelligence requires learning environments that foster reasoning, justification, and real-world problem-solving. In this study, the Euclidean Algorithm served not only as a problem-solving tool but also as a mediator for constructing mathematical meaning, enabling a transition from technical to professional knowledge.

In the final self-assessment guide, several prospectives teachers reported that they "understood the logic of the GCD and LCM for the first time" or that the experience transformed how they would approach teaching fractions. This finding reveals that affective and metacognitive components played a crucial role in the learning process. The structure of the sessions—centered on guided practice, meaningful problem-solving, and collaborative knowledge reconstruction—not only facilitated conceptual understanding but also triggered a deeper shift in attitudes and epistemological conceptions of mathematics. This transformation illustrates that traditional, limiting representations can be effectively challenged through structured, reflective, and contextualized instructional experiences.

From a more theoretical didactic perspective, the findings resonate with those of Etchegaray (2017), who identified tensions between institutional and personal meanings in teaching divisibility. In our study, the implementation of guided practice enabled the reconciliation of these meanings, allowing space for both the institutionalized algorithm and its personal reconstruction through pseudocode and visual representations—thereby breaking the rigidity of traditional didactic transpositions.

Furthermore, Climent, Carrillo, & Contreras (n. d.) emphasize the challenge of translating teachers' beliefs into actual classroom practices, even when conceptual clarity exists. The fact that most prospectives mathematics teachers in this study were able to justify the use of the algorithm and propose applications for lower secondary mathematics classrooms provides evidence of a real transformation in their teaching schemas. This supports the idea that reflective scaffolding and well-structured instructional experiences can help bridge the gap between beliefs and practices.

Finally, these results offer empirical support for the use of the DMKC framework as an integrative tool in designing contextualized instructional interventions—where theory, practice, emotion, culture, and technology are intertwined. The incorporation of computational thinking into mathemathics teacher preparation for lower secondary teaching should not be limited to digital tools but should also include logical structures such as the Euclidean Algorithm, which promote deep understanding, reasoning, and the pedagogical transfer of mathematica knowledge.

Building upon these insights, the outcomes of this study can also be interpreted through the lens of the extended TPACK-XK framework and the Theory of Instrumental Genesis (TIG), which highlight how the integration of technological tools, pedagogical strategies, and mathematical knowledge transforms both teaching practices and professional development. The participants' progression reflects not only an improvement in conceptual understanding, but also the emergence of didactic and instrumental competences that align with the principles of the Didactic-Mathematical Knowledge and Competences (DMKC) model.

In this sense, the experience reaffirms the importance of providing teachers with reflective tools that allow them to analyse and redesign their own practices critically and inclusively. The CID–DUA guide exemplifies such a tool, as it not only responds to curricular demands for inclusive education, but also strengthens reflective teaching practices by offering a structure that blends inclusivity with didactic rigor (Sánchez, Ledezma, & Font, 2025). These results will inform the redesign of future training cycles and strengthen the articulation between theoretically grounded instructional design and classroom realities in mathematics teacher preparation.

6. Conclusion

This study answers the initial research question affirmatively: activities centered on rediscovering the Euclidean Algorithm and applying it to the LCM significantly enhance both conceptual understanding and didactic-mathematical competence among prospectives teachers. The five-session instructional strategy facilitated a transition from fragmented knowledge and mechanical strategies to a more structured, logical, and contextual understanding of mathematical concepts. The use of pseudocode served as a bridge between mathematical and computational thinking. The observed quantitative and qualitative progress suggests that the participants not only learned the mathematical content but also developed a renewed disposition toward mathematics instruction—more critical, reflective, and contextualized. This is essential to counteract the low national and international achievement levels.

Nevertheless, challenges remain, particularly in internalizing the division-based version of the Euclidean Algorithm and translating cognitive strategies into structured language. This underscores the need for additional guided practice sessions or the integration of digital simulations.

In this regard, the present study contributes to a broader reflection on the need to revise current teaching methodologies in mathematics education. In an era where traditional methods prove insufficient, alternative approaches are required to optimize learning. The instructional design presented here responds to that demand by promoting learning through problem-solving, collaborative work, logical reasoning, and the construction of applicable knowledge. As a result, students move beyond memorizing techniques to understanding, deducing, testing, and collectively constructing mathematical meaning with formative value.

The experience also demonstrates the potential for designing activities that foster intrinsic motivation toward mathematical content—especially in topics typically perceived as dry, such as GCD and LCM. This motivation was reinforced through the use of contextualized materials, relatable examples, and a focus on everyday problems.

In line with these results, and as part of a broader research agenda, this pilot experience is serving as the foundation for the redesign of training cycles to be implemented within a blended specialization program in mathematics education, with a duration of two semesters and 24 academic credits. This specialization will integrate the principles of the Didactic-Mathematical Knowledge and Competences (DMKC) model, reflective tools such as the CID–DUA guide (Sánchez, Ledezma, & Font, 2025), and educational technologies articulated through the extended TPACK-XK framework, with the goal of preparing teachers capable of planning, analysing, and redesigning their teaching practices in real-world contexts.

In line with these results, this pilot experience provides evidence for the redesign of formative cycles articulated with the EMAS training farmwork ando fot the strength of mathematics teacher preparation in panama, particularly in relation to lower secondary teaching context.

Future research could explore variations of this intervention by integrating interactive digital tools to strengthen the connection between computational thinking and the teaching of classical algorithms. Systematizing and replicating this experience in teacher preparation programs such as EMAS could yield significant transformations in mathematics education within the Panamanian school system. The integration of historical algorithms, computational thinking, and contextualized strategies has the potential to develop teachers capable of meaningfully connecting mathematical theory, practice, and culture. This approach not only enhances understanding of fundamental content but also fosters a critical and creative teaching mindset aligned with the current needs of the national education system.

Aligned with the broader objectives of the doctoral project, these findings do not seek to analyze teachers' didactic knowledge exhaustively. Instead, they establish an empirical basis for understanding how initial levels of conceptual and pedagogical understanding are developed around key mathematical structures. This provides a foundation for future research to analyze functional thinking and the application of integrated didactic evaluation models. Consequently, this study offers a conceptual starting point for further exploration of mathematics teaching practices, progressively incorporating insights from the Onto-Semiotic Approach, the Didactic–Mathematical Knowledge model, and technology-based pedagogical frameworks within authentic educational contexts.

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8. Author Contribution

Conceptualization, K.CH .and J.G.G.; methodology, L.M.M.; software, R.D.G and K.CH.; validation, L.M.M., J.G.G. and R.D.G.; formal analysis, K.CH..; investigation, J.G.G. and K.CH.; resources, L.M.M.; data curation, K.CH. and R.D.G.; writing—original draft preparation, L.M.M.; writing—review and editing, J.G.G. and K.CH.; visualization, R.D.G.; supervision, L.M.M.; project administration, L.M.M.; funding acquisition, K.CH.. All authors have read and agreed to the published version of the manuscript.

References

Climent, N., Carrillo, J., & Contreras, L. C. (s. f.). La difícil traslación de las creencias del profesor al sentido de la asignatura: El caso de Matemáticas y su Didáctica en Educación Primaria. *Departamento de Didáctica de las Ciencias, Universidad de Huelva*. https://dialnet.unirioja.es/descarga/articulo/8033620.pdf

Etchegaray, S. C. (2017). Análisis de significados personales e institucionales sobre prácticas de divisibilidad en el marco del enfoque ontosemiótico. En J. M. Contreras, P. Arteaga, G. R. Cañadas, M. M. Gea, B. Giacomone y M. M. López-Martín (Eds.), *Actas del Segundo Congreso Internacional Virtual sobre el Enfoque Ontosemiótico del Conocimiento y la Instrucción Matemáticos*.

Fauzan, A., Yerizon, Y., & Yulianti, D. (2020). The RME-based local instructional theory for teaching LCM and GCF in primary school. *Journal of Physics: Conference Series*, *1554*(1), 012078. https://doi.org/10.1088/1742-6596/1554/1/012078

Godino, J. D. (2013). Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas. *Cuadernos*, 11, 111–132.

Guía para el Diseño de un Artículo Científico: Enseñanza del MCM con el Algoritmo de Euclides. (2024). Documento técnico no publicado. Universidad de Panamá.

Li, H.-C., & Tsai, T.-L. (2022). The effects of a problem-based learning intervention on primary students' performance on greatest common factor and least common multiple and on their attitudes towards mathematics. *International Journal of Innovation and Learning*, *31*(1), 51–69. https://doi.org/10.1504/IJIL.2022.119636

López Rodríguez, F. (2002). *La resolución de problemas en matemáticas: Teoría y experiencias*. Graó.

Nordby, S. K., Bjerke, A. H., & Mifsud, L. (2022). Computational thinking in the primary mathematics classroom: A systematic review. *Digital Experiences in Mathematics Education*, *8*(1), 27–49. https://doi.org/10.1007/s40751-022-00102-5

OECD. (2023). *PISA 2022 results (Volume I): The state of learning and equity in education*. PISA, OECD Publishing. https://doi.org/10.1787/53f23881-en

Panou, E., & Violetis, A. (2018). Teaching astronomy using monuments of cultural heritage: The educational example of "Horologion of Andronikos Kyrrhestes." *Scientific Culture*, *4*(2), 77–83.

Pino-Fan, L. R., Castro, W. F., & Font Moll, V. (2023). A Macro Tool to Characterize and Develop Key Competencies for the Mathematics Teacher's Practice. *International Journal of Science and Mathematics Education*, *21*, 1407–1432. https://doi.org/10.1007/s10763-022-10301-6

Pino-Fan, L. R., Godino, J. D., & Font, V. (2018). Assessing key epistemic features of didactic-mathematical knowledge of prospective teachers: The case of the derivative. *Journal of Mathematics Teacher Education, 21*, 63–94. https://doi.org/10.1007/s10857-016-9349-8

Rodríguez-Martínez, J. A., González-Calero, J. A., & Sáez-López, J. M. (2020). Computational thinking and mathematics using Scratch: an experiment with sixth-grade students. *Interactive Learning Environments*, *28*(3), 316–327. https://doi.org/10.1080/10494820.2019.1612448

Sabrigiriraj, M. (2024). A tutorial on new methods and algorithms for solving LCM and GCD. *WSEAS Transactions on Computers, 23*, 136–143. https://doi.org/10.37394/23205.2024.23.11

Sánchez, A., Ledezma, C., & Font, V. (2025). A Proposal of Integration of Universal Design for Learning and Didactic Suitability Criteria. *Education Sciences*, *15*(7), 909. https://doi.org/10.3390/educsci15070909

Segarra, L. (1999). Juegos y matemáticas. *Aula de Innovación Educativa*, 78, 25–28.

Socas, M., & Camacho, M. (2003). Conocimiento matemático y enseñanza de las matemáticas en la educación secundaria: Algunas reflexiones. *Boletín de la Asociación Matemática Venezolana*, *X*(2), 151–172.

Suarsana, I. M. (2021). Developing Interactive Digital Mathematics Book with Multi Representation Approach for Deaf Students. *International Journal of Emerging Technologies in Learning (iJET)*, *16*(13), 128–141. https://doi.org/10.3991/ijet.v16i13.22459

Valbuena-Duarte, S., Padilla Escorcia, I. A., & Rodríguez Bossio, E. (2021). Reconocer la inteligencia lógico-matemática en estudiantes con capacidades excepcionales. *Tecné, Episteme Y Didaxis: TED*, 49, 53–72. https://doi.org/10.17227/ted.num49-8152