Mathematical Modeling Competency of Pre-service Mathematics Teachers: Gap Diagnosis, Influencing Factors, and Cultivation Pathways

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ABSTRACT:

Purpose: This study aims to identify gaps in the mathematical modeling competency of pre-service mathematics teachers, explore key influencing factors, and propose evidence-based cultivation pathways.

Methodology: A mixed-methods design was adopted. A questionnaire survey was administered to 305 pre-service mathematics teachers from six teacher education institutions in Sichuan Province, China, and semi-structured interviews were conducted with 21 teacher educators. Data were analyzed using the Modified Priority Needs Index (PNI), multiple regression analysis, and thematic analysis.

Main Findings: Based on evaluations by 67 teacher educators, the most critical competency gaps were algorithm implementation (PNI = 0.50), applicability analysis (PNI = 0.43), and interpretation of meaning (PNI = 0.34). Multiple regression analysis of the 305 pre-service teachers' self-reports showed that critical thinking ($\beta = 0.349$, $p < 0.001$), practice opportunities ($\beta = 0.225$, $p < 0.001$), and evaluation orientation ($\beta = 0.214$, $p < 0.001$) were the strongest correlates of self-rated modeling competency among the variables examined, with the model accounting for 64.9% of the variance (adjusted $R^2 = 0.649$). Learning motivation, metacognition, and curriculum structure were also significantly associated with modeling competency ($\beta = 0.110$–0.158, $p < 0.01$).

Keywords: Mathematical modeling competency; Pre-service mathematics teachers; Modified Priority Needs Index (PNI); Critical thinking; Needs assessment.

1. Introduction

Mathematical modeling bridges mathematical theory and the real world. It has become a key element of core competency systems in 21st-century mathematics education reforms worldwide (Lu & Kaiser, 2022; Suh et al., 2021). In the field of teacher education, the modeling competency of pre-service mathematics teachers holds dual significance: it is both a component of their mathematical professionalism and is considered a factor influencing the quality of future mathematics modeling instruction (Blum & Leiß, 2005; Kaiser & Stender, 2013). Numerous studies have shown that mathematical modeling is a challenging task for many pre-service teachers (Anhalt & Cortez, 2016; Cirillo et al., 2016).

In recent years, the international academic community has examined the mathematical modeling competency of pre-service mathematics teachers from multiple perspectives. Research has shown that the mathematical modeling competency of pre-service mathematics teachers is associated with factors such as critical thinking and metacognitive awareness (Kannadass et al., 2023; Oficiar et al., 2024). Concurrently, research has revealed that pre-service mathematics teachers commonly encounter difficulties in mathematizing real-world situations, selecting appropriate models, implementing algorithms, and validating results (Bukova-Güzel, 2011; Kannadass et al., 2023; Haines & Crouch, 2007). To address these difficulties, existing studies have demonstrated that authentic modeling tasks, collaborative learning, and modeling decision tools can effectively enhance pre-service teachers' modeling competency (Wickstrom & Jung, 2024). Furthermore, the international academic community has accumulated substantial findings concerning the educational value of modeling, teacher professional development, and cross-cultural comparative studies (e.g., Taite & DiNapoli, 2025).

Currently, mathematics teacher education programs at Chinese normal universities widely offer mathematical modeling courses, with the core objective of cultivating pre-service mathematics teachers' ability to solve real-world problems using mathematical methods. However, existing research on Chinese pre-service mathematics teachers' mathematical modeling competency still has several limitations. For example, (1) most studies provide only holistic descriptions, lacking quantitative diagnoses of competency gaps across sub-dimensions of modeling ability (e.g., Formulation, Solving, Interpretation, Verification), and thus cannot answer the questions of "which specific aspect is the weakest" or "what the priority order for improvement should be"; (2) the effects of internal psychological factors (e.g., critical thinking, metacognition, learning motivation) and external environmental factors (e.g., practice opportunities, evaluation orientation, curriculum structure) on pre-service mathematics teachers' mathematical modeling competency have not been systematically examined; and (3) there is a lack of systematic research on cultivation pathways for pre-service mathematics teachers' mathematical modeling competency tailored to the Chinese local context.

To this end, this study constructs a three-tier theoretical framework (influential factors layer → skill layer → application layer) to explain the hierarchical structure of pre-service mathematics teachers' mathematical modeling competency development, and investigates the following three research questions (RQ):

RQ1: From the perspective of teacher educators, which dimensions and sub-dimensions exhibit gaps requiring priority improvement between desirable level and current level in the process of cultivating pre-service mathematics teachers' mathematical modeling competency?

RQ2: What is the mechanism through which internal psychological factors (critical thinking, metacognition, learning motivation) and external environmental factors (practice opportunities, evaluation orientation, curriculum structure) influence pre-service mathematics teachers' mathematical modeling competency?

RQ3: Based on the identified gaps and influencing factors, what evidence-based cultivation pathways can be proposed to enhance pre-service mathematics teachers' modeling competency in the Chinese context?

2. Literature Review

- Construct Definition: Process Competencies of Mathematical Modeling

Based on the conceptualization of mathematical modeling competency (Blomhøj & Jensen, 2003) and the widely adopted process framework of mathematical modeling instruction in Chinese undergraduate teacher education institutions, this study operationalizes mathematical modeling competency into four process competencies: Formulation, Solving, Interpretation, and Verification. These four processes constitute a complete modeling cycle that moves from real-world problems to mathematical models and back to real-world contexts. This framework is consistent with the modeling cycles proposed by scholars such as Blum & Leiß (2005) and Ferri (2006).

Formulation is defined as the inductive process of transforming ill-structured real-world problems into structured mathematical models (Blum & Leiß, 2005; Ferri, 2006). This study operationalizes formulation into three sub-dimensions: variable identification, relationship construction, and model selection.

Solving refers to the cognitive-operational process of deriving solutions from mathematical models through deductive reasoning and computational methods (Cevikbas et al., 2022; Dong et al., 2024). This study operationalizes solving into three sub-dimensions: algorithm design, algorithm implementation, and computational optimization.

Interpretation is the cognitive process of mapping abstract mathematical solutions back onto original real-world contexts, thereby endowing them with practical significance (Blum & Leiß, 2005; Ferri, 2006). This study operationalizes interpretation into three sub-dimensions: explanation of results, interpretation of meaning, and application guidance.

Verification is the process of assessing the consistency between model solutions and initial realities, and examining the robustness of the model under different conditions (Galbraith et al., 2013; Zbiek & Conner, 2006). This study operationalizes verification into three sub-dimensions: validation testing, applicability analysis, and improvement suggestions.

- Classification of Influencing Factors: Internal Psychological and External Environmental Factors
- Internal Psychological Factors

Critical thinking refers to the ability to abstract problems, reason logically, and test hypotheses (Suh et al., 2017). In mathematical modeling, it is considered a core cognitive resource for problem representation, strategy selection, and result evaluation (Lu & Kaiser, 2022). Existing research indicates that critical thinking is not only significantly correlated with modeling performance but also positively associated with the development of complementary skills such as computational thinking (Kannadass et al., 2023). This study operationalizes critical thinking into three sub-dimensions: problem abstraction, logical reasoning, and hypothesis testing.

Metacognition involves an individual's planning, monitoring, and regulation of their own cognitive activities (Stillman & Galbraith, 1998; Stillman, 2011). According to Pintrich's self-regulated learning theory (Pintrich, 2000), metacognition is an indispensable higher-order regulatory mechanism in complex problem solving. It is believed to support learners in self-monitoring and adjusting strategies at each stage of modeling.

Learning motivation, as an internal affective driver, is included as an internal psychological factor based on self-determination theory (Ryan & Deci, 2000). This theory posits that intrinsic motivation and autonomy-supportive evaluation significantly promote sustained learning engagement. In mathematical modeling tasks that require continuous effort and overcoming difficulties, the role of learning motivation is particularly critical.

- External Environmental Factors

Practice opportunities refer to the extent to which students engage in authentic, open-ended modeling problems within mathematical modeling courses, thereby experiencing the complete modeling process. Drawing on Vygotsky's (1978) sociocultural theory and the concept of scaffolding (Wood et al., 1976), learners who participate in challenging, authentic tasks with the support of more capable peers or instructors are able to effectively construct knowledge and skills within their zone of proximal development. Wickstrom and Jung

(2024) provided empirical support for this perspective in mathematics modeling education, demonstrating that authentic modeling tasks offer pre-service teachers critical opportunities to understand the modeling process, develop modeling skills, and build awareness of instructional practices.

Evaluation orientation influences the depth of pre-service teachers' engagement in the modeling process and the development of their professional competency. Research indicates that training on an assessment scheme based on the sub-phases of the modeling process (e.g., problem understanding, simplification and assumption making, mathematisation, solving and interpreting, validation, and process documentation) significantly enhances pre-service teachers' ability to diagnose and evaluate students' modeling solutions (Ferri & Greefrath, 2026). Moreover, through formative assessment strategies such as collaboratively constructing rubrics and receiving instructor feedback, pre-service teachers internalize a systematic understanding of classroom management, mathematics teaching, problem solving, and modeling instruction within their lesson plans (Villa-Ochoa et al., 2021). In conjunction with Besser et al.'s (2015) study on in-service teachers, training focused on formative feedback significantly strengthens teachers' pedagogical content knowledge (PCK) for handling modeling tasks. Therefore, evaluation orientation not only improves pre-service teachers' immediate learning outcomes but also shapes the development of their future competency in teaching modeling. This influence can be further explained by the role of self-assessment in promoting self-regulated learning (Panadero, Jonsson, & Botella, 2017). Their meta-analysis demonstrated that systematic self-assessment training significantly enhances students' use of metacognitive strategies, which are a core component of self-regulated learning, thereby transforming external evaluation into internal monitoring and fostering autonomous learning and deeper competency development.

Curriculum structure refers to the systematic arrangement of course objectives, the sequencing of learning modules, the selection of content, and the organization of learning tasks. Unlike practice opportunities, which focus on the frequency and accessibility of hands-on modeling activities, curriculum structure emphasizes the overall design and logical flow of the mathematical modeling course. According to self-determination theory (Ryan & Deci, 2000), when curriculum structure satisfies learners' three basic psychological needs (autonomy, competency, and relatedness), students are more likely to develop higher intrinsic motivation and engagement. A meta-analysis by Howard et al. (2021) further demonstrated that autonomous motivation is significantly associated with students' academic performance and engagement, thereby providing empirical support for this pathway.

- Theoretical Framework: Integration of the Three-Tier Transmission Logic
- Theoretical Foundations

This study integrates three complementary theoretical perspectives. Bloom's Taxonomy of Educational Objectives (Anderson & Krathwohl, 2001) provides a hierarchical model of cognitive abilities, indicating that higher-order abilities depend on lower-order ones. In the present framework, this principle directly applies to the progression from the skill layer to the application layer. The skill layer consists of four foundational modeling competencies, namely Formulation, Solving, Interpretation, and Verification, each with three sub-dimensions. The application layer comprises higher-order integrated competencies, including solving complex problems, cross-contextual transfer, creative modeling, and reflective practice. This taxonomy supports the view that mastery of the skill-layer competencies is a prerequisite for achieving the application-layer outcomes.

Self-Determination Theory (SDT) (Ryan & Deci, 2000) explains how intrinsic motivation and supportive evaluation foster autonomy, competency, and relatedness, thereby supporting the development of complex competencies. In this study, SDT provides the theoretical basis for understanding how learning motivation is positively related to pre-service teachers' engagement and progress from the skill layer to the application layer. By satisfying learners' basic psychological needs, motivation enhances sustained effort and deeper learning in modeling tasks.

Vygotsky's Sociocultural Theory (Vygotsky, 1978) emphasizes that cognitive development is rooted in social interaction and the use of cultural tools. Its concept of the zone of proximal development (ZPD) indicates that a scaffolded practical environment can effectively bridge the gap between current and potential competency levels (Wood et al., 1976). This theory underpins the role of external environmental factors, namely practice opportunities, evaluation orientation, and curriculum structure, in facilitating pre-service teachers' transition from the skill layer to the application layer. Through appropriate scaffolding, such as authentic tasks, formative feedback, and peer collaboration, these external factors create conditions that enable learners to achieve higher levels of modeling competency.

Based on the above theoretical foundations, this study constructs a three-tier competency development framework as illustrated in Figure 1: Influential Factors Layer → Skill Layer → Application Layer. This framework directly corresponds to the three research questions.

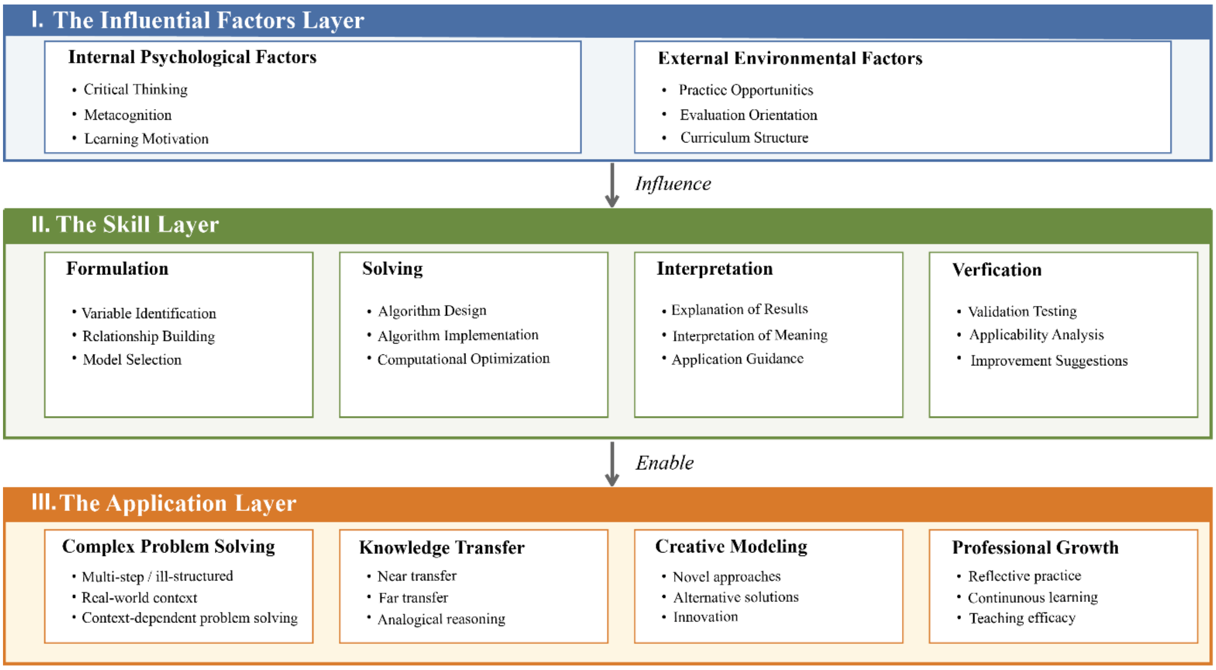

Figure 1. The Three-Tier Theoretical Framework of Mathematical Modeling Competency

Note: Arrows indicate hypothesized relationships: internal psychological factors and external environmental factors are hypothesized to be associated with the development of skill-layer competencies, which in turn enable application-level professional growth.

3. Research Methods

- Participants and Data Collection

The study participants included pre-service mathematics teachers and their instructors from the Mathematics and Applied Mathematics program at six undergraduate normal universities in Sichuan Province. Data collection was conducted in phases from September to October 2025.

Self-reported sample of pre-service mathematics teachers: A stratified random sampling by grade level (first to fourth year) was adopted, with the sample size for each grade proportional to the actual number of students. Data were collected via the Questionnaire Star platform, yielding 305 valid questionnaires (valid response rate: 94.7%). The sample distribution was as follows: 38 first-year students (12.5%), 74 second-year students (24.3%), 103 third-year students (33.8%), and 90 fourth-year students (29.5%); 144 male (47.2%) and 161 female (52.8%).

Teacher Educator Evaluation Sample. Using institution-based stratified random sampling, and based on the actual number of mathematics teacher educators at six normal universities, quotas were allocated in proportion to the number of teacher educators at each institution. A total of 67 teacher educators were invited to participate in the evaluation via online platforms (WeChat, QQ, and Questionnaire Star), all of whom completed valid questionnaires (valid response rate: 100%). The invited teacher educators rated both the current and expected levels of pre-service mathematics teachers on a 5-point scale, according to the expectations for modeling competency specified in the national mathematics curriculum standards and teacher education curricula. These ratings were used to calculate the Modified Priority Needs Index (PNI) to diagnose competency gaps. Subsequently, based on stratification by institution type and years of teaching experience, 21 teacher educators who had completed the evaluation were selected to participate in semi-structured interviews, aiming to gain an in-depth understanding of the factors influencing modeling competency.

- Research Instruments

Pre-service Mathematics Teachers' Self-Report Scale of Mathematical Modeling Competency. Based on the Guidelines for Assessment and Instruction in Mathematical Modeling Education (Bliss et al., 2016) and adapted to the Chinese teacher education context, a self-report questionnaire was developed. The scale comprises four core process competencies (Formulation, Solving, Interpretation, Verification), each with three sub-dimensions. In addition, the questionnaire includes six predictor variables: critical thinking (three sub-dimensions: problem abstraction, logical reasoning, hypothesis testing), practice opportunities, evaluation orientation, curriculum structure, metacognition, and learning motivation. All items are rated on a five-point Likert scale (1 = strongly disagree, 5 = strongly agree). This scale is used to assess pre-service teachers' self-rated modeling competency levels and influencing factors, providing data for descriptive statistics, grade-level difference analyses, and multiple regression analysis.

Five experts (two in mathematics education, two in educational measurement, and one in computer science) assessed content validity using the Index of Item-Objective Congruence (IOC). IOC values for all items ranged from 0.80 to 1.00, with a mean of 0.87, indicating good content validity.

Teacher Educator Evaluation Scale. This scale serves as a parallel measure to the Pre-service Mathematics Teachers' Mathematical Modeling Competency Self-Report Scale, employing the same evaluation dimensions (four process competencies and their 12 sub-dimensions) and the same five-point Likert scale (1 = very low, 5 = very high), with only the item wording adjusted accordingly based on the evaluation perspective (other-rated vs. Self-rated).

Teacher Educator Interview Outline: A semi-structured design was adopted. The interview content focused on skill-level gaps, influencing factors, and application-level difficulties, aiming to provide qualitative data for thematic analysis and a reference basis for designing cultivation pathways. Subsequently, five experts reviewed the outline for relevance, comprehensiveness, and clarity and provided feedback. Based on the experts' feedback, two questions were revised and one question concerning external resources was added. The revised outline was approved by all experts.

- Data Analysis Methods

Quantitative data analysis. First, the Modified Priority Needs Index (PNI) was used to quantify the priority for improving pre-service mathematics teachers' modeling competency (Wongwanich, 2015). The formula is:

$$PNI = \frac{(I - D)}{D}$$

where I = mean desirable level, D = mean current level. The advantages of this index are twofold: (1) it employs relative rather than absolute discrepancies, thereby avoiding confounding effects due to variations in scale ranges across dimensions; and (2) its values are comparable across dimensions, facilitating direct prioritization of improvements, with a higher PNI value indicating a more urgent need for improvement.

Subsequently, multiple linear regression analysis (forced entry method) was used to examine the strength of the relationship between six predictor variables (critical thinking, metacognition, learning motivation, practice opportunities, evaluation orientation, curriculum structure) and pre-service mathematics teachers' modeling competency. Prior to the regression analysis, assumption checks were conducted for normality of residuals (Shapiro-Wilk test), homoscedasticity (Breusch-Pagan test), multicollinearity (variance inflation factor, VIF), and influential cases (Cook's distance).

Qualitative data analysis. Interview transcripts were analyzed using Braun and Clarke's (2006) six-phase thematic analysis. The first author performed open coding and category development, and an expert in applied mathematics was invited to code 20% of the interview transcripts for verification. The two researchers discussed the coding classifications until consensus was reached, with an inter-coder agreement of 92%. According to Guest et al.'s (2006) criterion, the sample of 21 participants reached information saturation in qualitative research (saturation point was observed after 12-15 interviews).

4. Research Results

- Reliability and Validity of the Measurement Instruments

Based on self-report data from 305 pre-service mathematics teachers, Cronbach's α for the four modeling process competencies ranged from 0.824 to 0.837, and α for each predictor variable ranged from 0.836 to 0.948, all exceeding the recommended threshold of 0.70 (Nunnally & Bernstein, 1994).

Confirmatory factor analysis (CFA) was conducted on 20 first-order latent variables, including 12 modeling sub-dimensions, three critical thinking sub-dimensions, and five single-factor predictors: practice opportunities, evaluation orientation, curriculum structure, metacognition, and learning motivation. The model demonstrated good fit (Hu & Bentler, 1999): $\chi^2/df = 1.19$, CFI = 0.972, RMSEA = 0.025, SRMR = 0.033. All standardized factor loadings ranged from 0.72 to 0.90 ($p < .001$), above the recommended threshold of 0.70 (Hair et al., 2014).

Composite reliability (CR) ranged from 0.791 to 0.948, and average variance extracted (AVE) ranged from 0.557 to 0.783, meeting the criteria of CR $\geq$ 0.70 and AVE $\geq$ 0.50, respectively (Bagozzi & Yi, 1988; Fornell & Larcker, 1981). Discriminant validity was supported, as the square root of the AVE for each latent variable was greater than its highest correlation with any other latent variable.

This study employed Harman's single-factor test to assess common method bias (Podsakoff et al., 2003). All items from the questionnaire were subjected to an unrotated principal component analysis. The results showed that the first unrotated factor explained 26.4% of the total variance, which is below the conventional threshold of 40%, indicating that common method bias is not a serious concern in this study.

- Teacher Educators' Perspective: Gap Diagnosis and Attribution Analysis
- PNI Analysis: Skill Layer Gap Diagnosis

Table 1 presents the evaluation results of 67 teacher educators across four dimensions (Formulation, Solving, Interpretation, Verification) and twelve sub-dimensions. Overall, the mean current level across sub-dimensions was 3.22, the mean desirable level was 3.86, and the overall PNI (based on the mean of sub-dimension means) was 0.20. In terms of modeling process competencies, PNI values ranged from 0.13 to 0.25, with Solving and Verification demonstrating the greatest need for improvement (both with PNI = 0.25). At the sub-dimension level, the three sub-dimensions with the largest gaps were identified as the highest priority for improvement, namely: algorithm implementation (PNI = 0.50), applicability analysis (PNI = 0.43), and interpretation of meaning (PNI = 0.34).

Table 1: Needs assessment of pre-service mathematics teachers' mathematical modeling competency (N = 67)

Dimension / Sub-dimension	Current (D)		Desirable (I)		PNI	Rank
	Mean	SD	Mean	SD		
Formulation	3.47	0.25	3.92	0.06	0.13	Ⅳ
Variable Identification	3.36	0.83	3.91	0.75	0.16	8
Relationship Building	3.76	0.74	3.99	0.77	0.06	11
Model Selection	3.30	0.85	3.87	0.81	0.17	7
Solving	3.12	0.40	3.89	0.15	0.25	Ⅰ
Algorithm Design	3.18	0.83	3.81	0.76	0.20	4
Algorithm Implementation	2.70	0.84	4.06	0.78	0.50	1
Computational Optimization	3.49	0.77	3.81	0.87	0.09	10
Interpretation	3.18	0.35	3.77	0.11	0.19	Ⅲ
Explanation of Results	3.07	0.91	3.66	0.83	0.19	6
Interpretation of Meaning	2.90	0.74	3.88	0.71	0.34	3
Application Guidance	3.57	0.86	3.76	0.78	0.05	12
Verification	3.11	0.29	3.88	0.11	0.25	Ⅱ
Validation Testing	3.19	0.84	3.81	0.72	0.19	5
Applicability Analysis	2.79	0.93	4.00	0.82	0.43	2
Improvement Suggestions	3.36	1.04	3.82	0.85	0.14	9
Overall	3.22	0.32	3.86	0.11	0.20	-

Note: [a] Dimension scores (Formulation, Solving, Interpretation, Verification) are aggregated means of their respective three sub-dimensions.

[b] Roman numerals (Ⅰ -Ⅳ) rank the four dimensions; Arabic numerals (1-12) rank all twelve sub-dimensions.

- Teacher Educator Interviews: Qualitative Analysis and Categorization of Influencing Factors

Based on semi-structured interviews with 21 teacher educators, this study categorized the manifestations and problems in the current development of pre-service mathematics teachers' mathematical modeling competency into the following three groups according to the nature of the content.

Category 1: Curriculum and instructional environment. (1) At present, mathematical modeling courses in normal universities are mostly elective, with teaching content centered on textbook cases and past problems from the National College Mathematical Modeling Competition, lacking problems drawn from students' real-life contexts. (2) Modeling practice relies mainly on the National College Mathematical Modeling Competition held in September each year, with few practice opportunities outside the competition. (3) The evaluation system primarily includes: daily performance (attendance, class participation, group discussion), phased assignments or reports, mid-term assessment (major assignment/short paper/defense), and final assessment (course paper/comprehensive modeling/defense). However, most students overemphasize achieving passing scores on the final exam or meeting course completion requirements, neglecting the process-oriented learning of modeling. (4) Modeling tools mainly consist of programming software such as MATLAB, Python, and LINGO. Many students rely on textbook or online open-source code, as well as AI-assisted programming tools (e.g., Doubao, DeepSeek), to execute model computations, and have difficulty completing programming tasks independently.

Category 2: Students' Foundational Cognition and Behaviors. (1) 66.7% of the teacher educators reported that students had difficulties in extracting key variables and constructing logical relationships. One teacher illustrated: "They can read a problem about analyzing factors affecting housing prices, but struggle to identify which variables are relevant (e.g., location, floor area, distance to subway stations) and how they are interrelated with housing prices" (Teacher 12). (2) 61.9% of the teacher educators pointed out that students lacked sustained engagement in modeling tasks, often treating them merely as course requirements rather than opportunities for professional growth. (3) 42.8% of the teacher educators observed that students stopped after obtaining computational results, rarely explaining the meaning of the results or analyzing the limitations of the models.

Category 3: Students' application-level performance. In terms of applying mathematical modeling competency, 71.4% of the teacher educators observed that the majority of students are deficient in authentic problem solving, cross-contextual transfer, reflective practice, and awareness of professional growth. For example, some teacher educators pointed out that students can complete calculations but do not know how to use the model for actual decision-making; they can solve population growth problems in biology but cannot transfer the same logic to economic scenarios; and they rarely take the initiative to reflect or consider how modeling knowledge might be used in future teaching.

Based on the above analysis and a review of the relevant literature (see Section 2.2), we identified six potential factors affecting pre-service teachers' mathematical modeling competency: internal psychological factors (critical thinking, metacognition, learning motivation) and external environmental factors (practice opportunities, evaluation orientation, curriculum structure).

- Students' Perspective: Testing of Influencing Factors

- Descriptive Statistics of Self-Rated Modeling Competency

The overall mean of students' self-rated mathematical modeling competency was 3.32 (SD = 0.70). The mean scores for each dimension were: Formulation 3.47 (SD = 0.79), Interpretation 3.36 (SD = 0.84), Solving 3.22 (SD = 0.89), and Verification 3.21 (SD = 0.85). The scores for Solving and Verification were relatively low, consistent with the PNI diagnosis results from the teacher educators.

- Multiple Regression Analysis: Examining the Associations of Potential Influencing Factors

Based on self-report data from 305 pre-service mathematics teachers, a multiple regression analysis was conducted with overall modeling competency as the dependent variable and critical thinking, practice opportunities, evaluation orientation, curriculum structure, metacognition, and learning motivation as independent variables. The model exhibited strong explanatory power, with $R^2 = 0.656$ and adjusted $R^2 = 0.649$.

The standardized regression coefficients (see Table 2) showed that critical thinking had the strongest association ($\beta = 0.349$, $p < 0.001$), followed by practice opportunities ($\beta = 0.225$, $p < 0.001$) and evaluation orientation ($\beta = 0.214$, $p < 0.001$); learning motivation ($\beta = 0.158$, $p < 0.001$), metacognition ($\beta = 0.124$, $p = 0.001$), and curriculum structure ($\beta = 0.110$, $p = 0.003$). These results aligned with the judgments of teacher educators: 66.7% of the teacher educators interviewed mentioned students' difficulties in logical reasoning and problem abstraction, which corroborates the finding that critical thinking had the largest β value.

All regression assumption checks were satisfied: normality of residuals (Shapiro-Wilk $p = 0.196$), homoscedasticity (Breusch-Pagan $p = 0.134$), and all VIF values were below 2 (no multicollinearity). Robustness checks indicated that the maximum Cook's distance was 0.035 (<1), with no influential cases; the effect size Cohen's $f^2 = 1.905$ (> 0.35), indicating high practical significance of the model. Residual diagnostic plots (see Figure 2) showed no obvious heteroscedasticity pattern in residuals against fitted values, and the points in the normal Q-Q plot closely followed the diagonal line, further supporting the linearity and residual normality assumptions.

Table 2: Regression analysis results of factors affecting mathematical modeling competency (N=305)

Predictor Variables	B	SE	β	t	p	95% CI	VIF
(Constant)	0.689	0.123	-	5.59	<.001	[0.447, 0.931]	-
Critical Thinking	0.270	0.034	0.349	7.90	<.001	[0.203, 0.337]	1.687
Metacognition	0.078	0.023	0.124	3.30	0.001	[0.032, 0.124]	1.222
Learning Motivation	0.115	0.028	0.158	4.14	<.001	[0.060, 0.170]	1.255
Practice Opportunities	0.116	0.020	0.225	5.77	<.001	[0.077, 0.155]	1.314
Evaluation Orientation	0.154	0.028	0.214	5.49	<.001	[0.099, 0.209]	1.313
Curriculum Structure	0.066	0.023	0.110	2.95	0.003	[0.021, 0.111]	1.207

Note: B = unstandardised coefficient; SE = standard error; β = standardised coefficient; CI = confidence interval; VIF = variance inflation factor

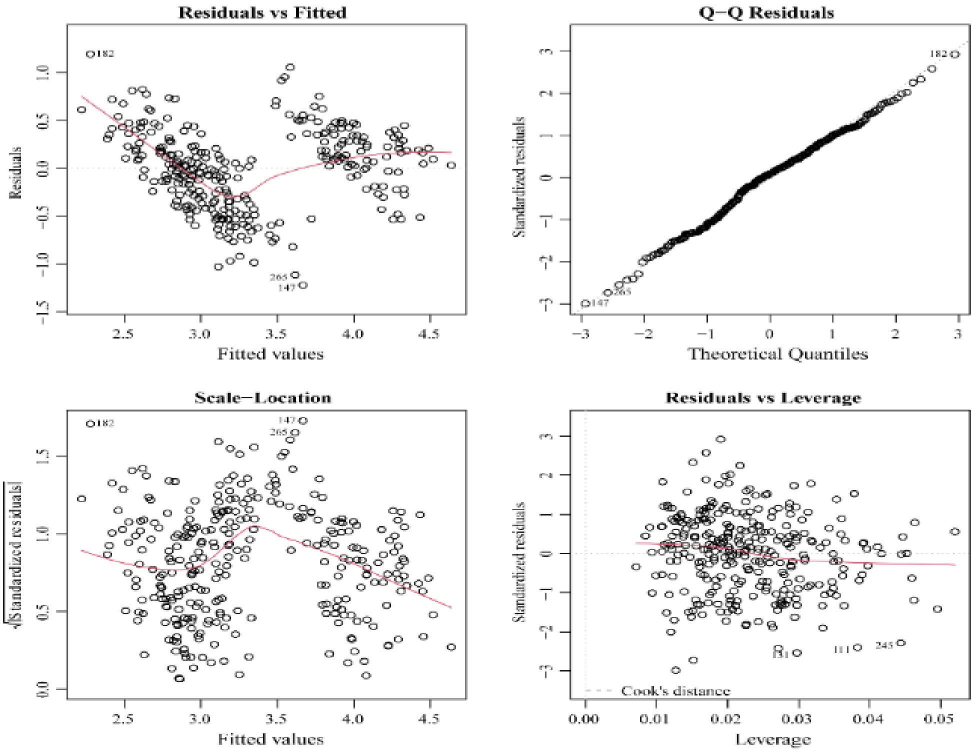

Figure 2. Regression diagnostic plots

- Grade differences analysis: A descriptive examination of mathematical modeling competency development

As a supplementary analysis, this study examined differences in mathematical modeling competency among students across different grades. One-way ANOVA revealed significant grade-level differences ($F(3,301) = 86.35$, $p < 0.001$, $\eta^2 = 0.463$), indicating that grade level explained 46.3% of the variance in modeling competency, representing a large effect size. Tukey HSD post-hoc tests showed that, except for the non-significant difference between first-year and second-year students ($p = 0.075$), all pairwise comparisons between other grades were significant ($p < 0.001$). The mean modeling competency scores exhibited an increasing trend from first year (2.64) to second year (2.88) to third year (3.32) and to fourth year (3.95), with the largest increase observed between the third and fourth years ($\Delta = 0.63$, $p < 0.001$). This increase may be associated with the accumulation of coursework and practical experiences (e.g., capstone projects, training for competitions, and teaching practicum), thereby providing a reference for the grade-differentiated cultivation of pre-service mathematics teachers' mathematical modeling competency.

Table 3: Analysis of Variance (ANOVA) Results for Mathematical Modeling Competency and Sub-Dimension Scores by Grade (N=305)

Grade	N	Overall	Formulation	Solving	Interpretation	Verification
Freshman	38	2.64 (0.21)	2.81 (0.47)	2.46 (0.49)	2.73 (0.54)	2.54 (0.50)
Sophomore	74	2.88 (0.24)	3.04 (0.56)	2.89 (0.61)	2.87 (0.64)	2.73 (0.51)
Junior	103	3.32 (0.56)	3.54 (0.71)	3.17 (0.78)	3.35 (0.75)	3.23 (0.75)
Senior	90	3.95 (0.68)	4.03 (0.73)	3.86 (0.93)	4.05 (0.67)	3.88 (0.81)

F (3, 301)	-	86.35	45.16	38.58	54.94	51.76
η^2	-	0.463	0.31	0.28	0.35	0.34

Note: Values are presented as mean (standard deviation). All F-tests have $p < 0.001$

5. Discussion

- Core Findings and Theoretical Interpretation

This study yielded three main empirical findings. (1) The most prominent gaps in modeling competence were observed in three subdimensions: algorithm implementation, applicability analysis, and interpretation of results; in terms of modeling process competencies, Solving and Verification were the two components with the highest need for improvement. (2) Critical thinking made the largest unique contribution to modeling competence, followed by practice opportunities and evaluation orientation; learning motivation, metacognition, and curriculum structure also had significant but relatively smaller predictive effects. Together, the six factors explained 64.9% of the variance. (3) Modeling competence showed a clear increasing trend with grade level, with the largest gain occurring between the third and fourth years.

The strongest predictive role of critical thinking is consistent with the finding of Kannadass et al. (2023) that computational thinking is positively associated with modeling competence. The significant effect of practice opportunities aligns with Vygotsky's sociocultural theory and the empirical work of Wickstrom and Jung (2024). The effect of evaluation orientation corroborates the findings of Panadero et al. (2017) regarding formative assessment promoting self-regulated learning and those of Ferri and Greefrath (2026) concerning assessment training enhancing assessment competence in mathematical modeling.

Although the direct predictive effect of curriculum structure on modeling competence was relatively weak, it remained statistically significant. From a theoretical perspective, a well-designed curriculum can stimulate intrinsic motivation by satisfying learners' needs for autonomy and competence, thereby promoting metacognitive engagement in the learning process (Ryan & Deci, 2000). Furthermore, the sample of this study was drawn from six normal universities in Sichuan Province, where the curricular designs of mathematical modeling courses are highly similar, which to some extent limited the observed variance of this variable.

The pronounced gap in the algorithm implementation subdimension echoes the concern raised by Stoyanova et al. (2026) that generative artificial intelligence may induce shallow learning. Relying on AI-generated code without understanding the underlying logic weakens debugging and interpretation skills, highlighting the need to adjust pedagogical guidance related to AI use.

- Evidence-Based Cultivation Pathways

Based on the above findings, this study proposes four tiered cultivation pathways, with specific measures designed separately for lower-grade (first and second year) and upper-grade (third and fourth year) pre-service mathematics teachers.

First, strengthen internal psychological factors. Given the significant predictive roles of critical thinking, learning motivation, and metacognition, and that most teacher educators indicated pre-service mathematics teachers experience difficulties in logical reasoning, lower-grade students could receive structured training in critical thinking using ill-structured real-world problems, along with autonomy-supportive motivational strategies (e.g., allowing free choice of problem contexts and setting phased challenges). Upper-grade students could maintain hypothesis-testing logs and project reflection reports, and incorporate modeling competitions and teaching practicum into the training process.

Teachers may also provide critical checklists (e.g., whether key variables are omitted, whether assumptions are reasonable) to help pre-service teachers systematically examine model applicability.

Second, address skill gaps. For the three highest-priority gaps, namely algorithm implementation, applicability analysis, and interpretation of results, lower-grade students could improve algorithm implementation through debugging logs, using AI tools only for code explanation and debugging suggestions, embedding reflective questions and sensitivity experiments in applicability analysis, and practicing interpreting results in authentic contexts. Upper-grade students could complete full-cycle modeling projects encompassing data collection, cleaning, and validation, and write corresponding reflection reports.

Third, optimize external environmental factors. Practice opportunities and evaluation orientation demonstrated strong predictive power, and curriculum structure also made a significant contribution. For lower grades, increase authentic practice opportunities (e.g., campus modeling days, modeling weeks, or intensive training for modeling competitions), adopt formative assessment and peer assessment, restructure course content using short-cycle, single-module project-based learning, and regularly conduct curriculum needs surveys to dynamically adjust teaching cases, training difficulty, and learning scaffolds based on pre-service teachers' feedback. For upper grades, raise evaluation standards by requiring complete models, research reports, and public defenses, and offer elective modules such as teaching practicum and advanced modeling methods.

Fourth, enhance application and transfer skills. Most teacher educators reported that pre-service teachers lack transfer ability, and applicability analysis is a critical weakness. Lower-grade students could engage in near-transfer training by applying the same model to similar contexts, using comparative analysis to help them develop a "one model, multiple contexts" cognitive schema, and incorporate modeling comparison analysis reports into assignments. Upper-grade students could be encouraged to engage in far transfer and creative modeling using ill-structured problems, integrate multiple mathematical methods (e.g., time-series analysis and cluster analysis), and transform modeling knowledge into lesson plans for simulated teaching.

- Research Limitations

This study has several limitations. First, the sample was drawn from six normal universities in Sichuan Province, so generalizability to other regions or types of institutions should be made with caution. Second, the cross-sectional design cannot establish causality; future longitudinal or experimental studies are needed to validate the proposed cultivation pathways. Third, although Harman' s single-factor test indicated no serious common method bias, self-report measures may still be subject to bias arising from respondents' tendency to provide answers that align with socially desirable expectations. Future research should employ more direct assessment approaches, such as rubric-based scoring of modeling tasks or portfolio assessment. Fourth, the qualitative interviews involved only 21 teacher educators, and this study did not include quantitative indicators for the "application-layer competencies" within the theoretical framework (e.g., cross-contextual transfer, creative modeling, reflective practice, and solving complex problems). Future research could expand the qualitative sample size to obtain richer phenomenological descriptions, while also developing standardized scales or task-transfer tests targeting application-layer competencies, thereby enabling a more comprehensive quantitative assessment of both application-layer and overall modeling competency.

6. Conclusion

Based on evaluations by teacher educators, self-reports by pre-service mathematics teachers, and semi-structured interviews, this study diagnosed the core gaps and key influencing factors in mathematical modeling competency among pre-service mathematics teachers in China. The findings support the proposed three-tier theoretical framework (influential factors layer → skill layer → application layer), indicating that internal psychological factors (critical thinking, metacognition, learning motivation) and external environmental factors (practice opportunities, evaluation orientation, curriculum structure) are significantly associated with skill-layer competencies (Formulation, Solving, Interpretation, Verification), thereby providing a theoretical pathway for further promoting application-level professional growth (e.g., cross-contextual transfer, creative modeling, reflective practice). Among the findings, algorithm implementation, applicability analysis, and interpretation of results emerged as the most prominent weaknesses, while critical thinking, practice opportunities, and evaluation orientation were the factors most strongly associated with modeling competence. Based on these findings, this study proposes evidence-based, grade-stratified cultivation pathways.

Future research should employ experimental or longitudinal designs to further validate the correlational findings of this study and develop direct assessment instruments targeting application-level competencies, so as to more comprehensively reveal the developmental patterns of preservice mathematics teachers' mathematical modeling competence and provide a scientific basis for cultivating excellent teachers capable of meeting the demands of future society.

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