

Stability Theory Corresponds to the Deformable Derivative

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ABSTRACT:

The fractional and generalized derivatives have gained more attention due to their broad and diverse applications. They offer great tool sets for modelling systems that exhibit dynamics with memory and heredity. A new operator was recently introduced, namely deformable derivative, which is capable of fractionalized derivatives. Stability theory is crucial in many areas of science and engineering. Stability theory shows how the solutions of differential equations behave under small perturbation. This communication addresses the relationship of stability theory with deformable derivative. Some basic properties of the new operator are proved and studied. Stability of systems of differential equations formed with the help of this operator is studied.

Keywords: Deformable Derivative; Stability Theory; Dynamical Systems; Lyapunov Stability; Generalized Calculus; Differential Equations; Eigenvalue Analysis; Fractional Calculus

1. Introduction:

The modern mathematics include the study of the differential equation. Many compounds in natural or human origin can be modeled using differential equations. Stability could depend on the properties of such a compound and vice versa. The stability theory deals with small disturbances in initial data leading to small disturbances in the subsequent reaction. This means that traits develop that counter a deviation from the main path. Since A. did this whole work. M. According to M. Lyapunov, important parts of theory.

Consider the classical differential equation

$$\frac{dx}{dt} = f(x, t)$$

where $x(t)$ represents the state variable and f is a sufficiently smooth function. Stability theory Stability is the ability of a solution to remain bounded researchers. Put differently, the solution converges to the equilibrium point with a small blow up. Asymptotic stability, helps whether solution will converge to an equilibrium point.

The **deformable derivative** is particularly interesting because it continuously deforms the classical derivative through a parameter α , where

$$0 \leq \alpha \leq 1$$

For a differentiable function $f(t)$, the deformable derivative is commonly defined by

$$D^\alpha f(t) = \beta f(t) + \alpha f'(t),$$

where

$$\beta = 1 - \alpha$$

Thus,

$$D^\alpha f(t) = (1 - \alpha)f(t) + \alpha \frac{df}{dt}$$

This definition establishes a smooth transition between the function itself and its ordinary derivative. Specifically,

- For $\alpha = 0$,

$$D^0 f(t) = f(t),$$

- For $\alpha = 1$,

$$D^1 f(t) = f'(t)$$

Hence, Let's consider an operator which combines or joins the static and dynamic behaviour of the system.

The response is the deformable derivative. We are observing the growing proximity of deformation derivatives in equations and expressions. It is reasonable to question the stability of the solutions. Consider the deformable differential equation

$$D^\alpha x(t) = F(x, t)$$

The equilibrium point x^* satisfies

$$F(x^*, t) = 0$$

A central objective is to determine conditions under which solutions satisfy

$$\lim_{t \rightarrow \infty} x(t) = x^*$$

The deformation parameter α introduces additional flexibility that may either enhance or weaken stability characteristics. Thus, the classical stability must be extended to a new dynamical system expressed by this deformable derivative. The study discusses how the stiff theory and deformable derivative are closely related. deformable dynamical

systems is theoretically performed. It draws conclusions on easily verifiable. In addition, the paper examines the effect of variation parameter on the system's behavior. Furthermore, it demonstrates that many classical results are recovered when $\alpha = 1$, thereby establishing consistency with conventional differential equation theory.

2. Literature Review:

In 2014, Khalil, Al Horani, Yousef, and Sababheh [1]. Their study offers notion simpler and intuitive alternative to other existing fractional derivatives. The proposed derivative also has many fundamental features of classical derivatives. Also, the author verified that the defined derivative satisfies linearity, product rule, and chain rules. This research is a significant step forward in generalized calculus and has sparked intense interest in investigating different fractional and deformable differential operators. In 2016, Benkhettou, Hassani and Torres [2] enlarged which unifies the continuous and discrete analysis within same mathematical frame. The authors have shown several results on this definition of calculus setting. To add to it, they have also proved their results useful in dynamical systems. The generalised calculus was extended and, as a subsequent development, stability dynamical which evolve on continuum and discrete time domains. Abdeljawad [3] carried out a comprehensive study on conformable fractional calculus in 2015 in which he obtained many results concerning conformable derivatives and integrals. Furthermore, this research is going to provide solid support for the newly introduced conformable fractional calculus. It will also prove interesting relations in generalized and ordinary differential operators.

Research by Pathak and Khan [4] in 2018 on deformable derivatives and differential equations. The authors developed a deformable derivative as an operator for making a smooth deformation of a function into its classical derivative, and applied deformable derivatives to some differential equations. According to them, this generalized derivative can also be used with generalized dynamical systems and certainly has the potential of generalized functions. This document can be considered as one of the greatest backbones of any

research article involving deformable derivatives and stability analysis. The consequences of the writers are an important contribution to the application of more generalized derivatives in mathematical modelling as well as differential equations of physical phenomena.

Scholarly article is cited in 2005. Khalil Grizzle's Nonlinear systems appeared in 2002. This work gives a detailed treatment of the equilibrium points, concepts stability, methods of Lyapunov and nonlinear dynamics. The explanation discusses the potential use of mathematical tools in achieving a system's analysis, on the one hand, and to explicitly obtain a solution from the relevant differential equations, on the other. The discussion also emphasizes the connections between the several notions of stability. Many physical systems have effectively applied the methods of stability and are becoming a fundamental reference to the modern stability theory. In 1961, Lyapunov's direct method and its applications were presented by LaSalle and Lefschetz [6]. Afterward, Different authors have used the technique of stability analysis.

The author examined the concept of Lyapunov stability for nonlinear autonomous systems in detail in study [6]. The stability analysis related to equilibrium points and control Lyapunov functions is studied. The author also implemented several Lyapunov function analyses applied to differently manufactured composite systems. This work changed stability theory. To this day, it remains one of the most popular methods for analyzing linear and nonlinear dynamical systems. In a study from 2015 [7], Khalil advanced the area of stability theory by examining nonlinear control systems. The system properties of Lyapunov stability, asymptotic stability, and exponential stability were remarked on robust control techniques. The writer also talked about the real problems in considering engineering applications stability. In addition, the author presented mathematical instruments that can be used directly in the study of the deformation differential system. In 2018, research [8] worked, while Strogatz published the second edition of his well-known book on nonlinear dynamics and chaos

The author covered equilibrium points, oscillations, bifurcations, and chaotic systems at a very intuitive

level. As noted by researchers, stability analysis is an essential part of a system's longtime behavior determination, especially nonlinear cases. Additionally, the author examines practical concerns regarding the nonlinear systems arising in physics, biology, chemistry, and engineering. Perko (2001) explained differential equations and dynamical systems in a detailed way with an emphasis on qualitative analysis.

Murray (2002) gave many biological examples for the application of differential equations and stability theory. Murray specifically illustrated the beauty of modeling in the description and quantitative analysis of population functions, ecology dynamics, epidemic spreads, and biological processes. The asymptotic nature of biological systems and conditions for stability of an equilibrium were derived heavily using the methods of stability theory. In 1999, Podlubny published a comprehensive book on fractional differential equations. This document offers a comprehensive theoretical framework for analytic techniques and engineering applications. Fractional differential equations were introduced as an important autonomous topic of fractional calculus. It displayed the utility memory and hereditary property systems. In their 2006 work, Oldham and Spanier showcased fractional calculus along with some applications.

This paper showed how defined. In addition, the fractional derivative as well as the fractional integration is proven to be useful. Numerous examples are provided by the authors to show that fractional operators appear naturally and are quite useful in solving problems science and engineering. Moreover, this work greatly contributes to mathematical physics and engineering sciences, having made fractional calculus popular among researchers. Kilbas, Srivastava and Trujillo [13] showed it in 2006.

Compared to classical differential operators, fractional operators are better suited for modelling various physical phenomena and engineering processes, due to the fact that they precisely capture memory behaviour of complex dynamical processes. Fractional differential equations model

complex dynamical processes that cannot be represented by classical equations.

Offer a more suitable option for mathematical modelling. Matignon studied stability properties of fractional differential equations and their control applications in 1996. He proved important stability criteria demonstrated that systems have different stability properties. This is a classical paper and the main reference on the stability properties generalized Lakshmikantham, Leela and Vasundhara Devi studied comprehensive. (2009) The study focused on the theory of solution, existence, uniqueness, boundedness and stability properties. The generalized derivative can be used effectively to describe systems which have memory nature and complex dynamical processor. On the whole, based on older works it is clear that Lyapunov stability and non-linear dynamics has been established and based on recent works it is clear that conformable and deformable derivatives has been introduced which are a good and flexible mathematical tool for modelling complex dynamical processes till now on generalised differential operators.

At this stage, the use of deformable derivatives can be very useful to understand equilibrium analysis, Lyapunov stability and dynamical system concepts. The paper seeks to provide a logical interpretation of these three concepts using a deformable derivative approach.

3. Preliminaries:

Deformable Derivative:

The **Deformable Derivative** is a generalized differential operator defined by

$$D^\alpha f(t) = \beta f(t) + \alpha f'(t),$$

where $\beta = 1 - \alpha$ and $0 \leq \alpha \leq 1$. It provides a smooth transition between a function and its classical derivative.

Stability Theory:

Stability Theory is Dynamical systems change or become self-deriving over time. They refer to the elements of mathematical sets which quantify a certain set of objects.

Dynamical Systems:

A **Dynamical System** is a mathematical model that describes the evolution of a system over time and is commonly represented by

$$\frac{dx}{dt} = f(x, t)$$

Lyapunov Stability:

An equilibrium point x^* is said to be **Lyapunov Stable** if every solution that starts sufficiently close to x^* remains close to x^* for all future time.

Fractional Calculus:

Fractional Calculus is the study of derivatives and integrals of arbitrary (non-integer) order. It extends classical calculus and is widely used for modeling memory and hereditary effects in dynamical systems.

4. Analysis of the Study:

Q.1: Find the deformable derivative of

$$f(t) = t^2 + 3t + 2$$

for

$$\alpha = \frac{1}{2}$$

Solution: The deformable derivative of a function $f(t)$ is defined as

$$D^\alpha f(t) = \beta f(t) + \alpha f'(t),$$

where

$$\beta = 1 - \alpha$$

Here,

$$\alpha = \frac{1}{2}$$

Therefore,

$$\beta = 1 - \frac{1}{2} = \frac{1}{2}$$

Now the given function is

$$f(t) = t^2 + 3t + 2$$

First, find the ordinary derivative of $f(t)$:

$$f'(t) = \frac{d}{dt}(t^2 + 3t + 2)$$

Using the rules of differentiation,

$$\frac{d}{dt}(t^2) = 2t,$$

$$\frac{d}{dt}(3t) = 3,$$

and

$$\frac{d}{dt}(2) = 0$$

Therefore,

$$f'(t) = 2t + 3$$

Now apply the formula of deformable derivative:

$$D^{\frac{1}{2}}f(t) = \frac{1}{2}f(t) + \frac{1}{2}f'(t)$$

Substitute the values of $f(t)$ and $f'(t)$:

$$D^{\frac{1}{2}}f(t) = \frac{1}{2}(t^2 + 3t + 2) + \frac{1}{2}(2t + 3)$$

Now simplify each term:

$$\frac{1}{2}(t^2 + 3t + 2) = \frac{t^2}{2} + \frac{3t}{2} + 1,$$

and

$$\frac{1}{2}(2t + 3) = t + \frac{3}{2}$$

Therefore,

$$D^{\frac{1}{2}}f(t) = \frac{t^2}{2} + \frac{3t}{2} + 1 + t + \frac{3}{2}$$

Combine the like terms:

$$\frac{3t}{2} + t = \frac{3t}{2} + \frac{2t}{2} = \frac{5t}{2},$$

and

$$1 + \frac{3}{2} = \frac{2}{2} + \frac{3}{2} = \frac{5}{2}$$

Hence,

$$D^{\frac{1}{2}}f(t) = \frac{t^2}{2} + \frac{5t}{2} + \frac{5}{2}$$

Therefore,

$$\boxed{D^{\frac{1}{2}}f(t) = \frac{1}{2}t^2 + \frac{5}{2}t + \frac{5}{2}}$$

Q.2: Find the deformable derivative of

$$f(t) = e^{2t}$$

for

$$\alpha = \frac{3}{4}$$

Solution: The deformable derivative is given by

$$D^\alpha f(t) = \beta f(t) + \alpha f'(t),$$

where

$$\beta = 1 - \alpha$$

Given,

$$\alpha = \frac{3}{4}$$

Therefore,

$$\beta = 1 - \frac{3}{4} = \frac{1}{4}$$

Now,

$$f(t) = e^{2t}$$

The ordinary derivative is

$$f'(t) = \frac{d}{dt}(e^{2t})$$

Using the chain rule,

$$\frac{d}{dt}(e^{2t}) = 2e^{2t}$$

Therefore,

$$f'(t) = 2e^{2t}$$

Now apply the deformable derivative formula:

$$D^{\frac{3}{4}}f(t) = \frac{1}{4}f(t) + \frac{3}{4}f'(t)$$

Substitute $f(t) = e^{2t}$ and $f'(t) = 2e^{2t}$:

$$D^{\frac{3}{4}}(e^{2t}) = \frac{1}{4}e^{2t} + \frac{3}{4}(2e^{2t})$$

Simplify:

$$D^{\frac{3}{4}}(e^{2t}) = \frac{1}{4}e^{2t} + \frac{6}{4}e^{2t}$$

$$D^{\frac{3}{4}}(e^{2t}) = \frac{7}{4}e^{2t}$$

Therefore,

$$\boxed{D^{\frac{3}{4}}(e^{2t}) = \frac{7}{4}e^{2t}}$$

Q.3: Find the deformable derivative of

$$f(t) = \sin t$$

for

$$\alpha = \frac{2}{3}$$

Solution: The deformable derivative is

$$D^\alpha f(t) = \beta f(t) + \alpha f'(t),$$

where

$$\beta = 1 - \alpha$$

Given,

$$\alpha = \frac{2}{3}$$

Therefore,

$$\beta = 1 - \frac{2}{3} = \frac{1}{3}$$

Now,

$$f(t) = \sin t$$

The ordinary derivative is

$$f'(t) = \cos t$$

Using the deformable derivative formula,

$$D^{\frac{2}{3}}(\sin t) = \frac{1}{3} \sin t + \frac{2}{3} \cos t$$

Therefore,

$$D^{\frac{2}{3}}(\sin t) = \frac{1}{3} \sin t + \frac{2}{3} \cos t$$

Q.4: Find the equilibrium point of the deformable differential equation

$$D^\alpha x(t) = -2x(t),$$

where

$$\alpha = \frac{1}{2}$$

Solution: The deformable derivative of $x(t)$ is

$$D^\alpha x(t) = \beta x(t) + \alpha x'(t)$$

Given,

$$\alpha = \frac{1}{2}$$

Hence,

$$\beta = 1 - \alpha = 1 - \frac{1}{2} = \frac{1}{2}$$

Now substitute this into the given differential equation:

$$D^{\frac{1}{2}}x(t) = -2x(t)$$

Using the formula,

$$\frac{1}{2}x(t) + \frac{1}{2}x'(t) = -2x(t)$$

Multiply the whole equation by 2:

$$x(t) + x'(t) = -4x(t)$$

Now shift $x(t)$ to the right side:

$$x'(t) = -4x(t) - x(t)$$

$$x'(t) = -5x(t)$$

For an equilibrium point, the state does not change with time. Therefore,

$$x'(t) = 0$$

So,

$$-5x = 0$$

Hence,

$$x = 0$$

Therefore, the equilibrium point is

$$x^* = 0$$

Q.5: Study the stability of the equilibrium point of

$$D^\alpha x(t) = -3x(t),$$

where

$$\alpha = \frac{1}{2}$$

Solution: The deformable derivative is given by

$$D^\alpha x(t) = \beta x(t) + \alpha x'(t)$$

Given,

$$\alpha = \frac{1}{2}$$

Therefore,

$$\beta = 1 - \frac{1}{2} = \frac{1}{2}$$

Now the given equation is

$$D^{\frac{1}{2}}x(t) = -3x(t)$$

Using the deformable derivative formula,

$$\frac{1}{2}x(t) + \frac{1}{2}x'(t) = -3x(t)$$

Multiply by 2:

$$x(t) + x'(t) = -6x(t)$$

Now simplify:

$$\begin{aligned}x'(t) &= -6x(t) - x(t) \\x'(t) &= -7x(t)\end{aligned}$$

This is a first-order linear differential equation. Its solution is

$$x(t) = Ce^{-7t},$$

where C is an arbitrary constant.

If the initial condition is

$$x(0) = x_0,$$

then

$$x(0) = Ce^0 = C$$

So,

$$C = x_0$$

Hence, the solution becomes

$$x(t) = x_0 e^{-7t}$$

Now examine the behavior as $t \rightarrow \infty$:

$$\lim_{t \rightarrow \infty} x(t) = \lim_{t \rightarrow \infty} x_0 e^{-7t}$$

Since

$$e^{-7t} \rightarrow 0 \text{ as } t \rightarrow \infty,$$

we get

$$\lim_{t \rightarrow \infty} x(t) = 0$$

Therefore, every solution starting near $x = 0$ approaches 0 as time increases.

Hence, the equilibrium point

$$x^* = 0$$

is

asymptotically stable

Also, since the convergence is of the form

$$|x(t)| = |x_0| e^{-7t},$$

the equilibrium point is

exponentially stable

Q.6: Using the Lyapunov function

$$V(x) = x^2,$$

study the stability of the system

$$D^\alpha x(t) = -kx(t),$$

where

$$k > 0, 0 < \alpha \leq 1$$

Solution: The given deformable differential equation is

$$D^\alpha x(t) = -kx(t)$$

Using the definition,

$$D^\alpha x(t) = \beta x(t) + \alpha x'(t),$$

where

$$\beta = 1 - \alpha$$

Thus,

$$\beta x(t) + \alpha x'(t) = -kx(t)$$

Now solve for $x'(t)$:

$$\alpha x'(t) = -kx(t) - \beta x(t)$$

$$\alpha x'(t) = -(k + \beta)x(t)$$

Therefore,

$$x'(t) = -\frac{k + \beta}{\alpha} x(t)$$

Since

$$k > 0, \beta \geq 0, \alpha > 0,$$

we have

$$\frac{k + \beta}{\alpha} > 0$$

Now consider the Lyapunov function

$$V(x) = x^2$$

Clearly,

$$V(0) = 0,$$

and

$$V(x) > 0 \text{ for } x \neq 0$$

Hence, $V(x)$ is positive definite.

Now compute its derivative along the trajectories:

$$\begin{aligned}\dot{V}(x) &= \frac{d}{dt}(x^2) \\ \dot{V}(x) &= 2xx'(t)\end{aligned}$$

Substitute

$$x'(t) = -\frac{k+\beta}{\alpha}x(t)$$

Then,

$$\dot{V}(x) = 2x\left(-\frac{k+\beta}{\alpha}x\right)$$

$$\dot{V}(x) = -\frac{2(k+\beta)}{\alpha}x^2$$

Since

$$\frac{2(k+\beta)}{\alpha} > 0,$$

we get

$$\dot{V}(x) < 0 \text{ for } x \neq 0$$

Therefore, $\dot{V}(x)$ is negative definite.

Hence, by Lyapunov stability theorem, the equilibrium point

$$x^* = 0$$

is

asymptotically stable

and because the system is linear with exponential decay, it is also

Exponentially Stable

5. Results and discussion:

The study investigates the relationship between stability theory and deformable derivative operator. The deformable derivative smooths out the transition from the identity operator to the classical

derivative via the deformation parameter α . This characteristic enables the formulation of generalized dynamical systems while preserving many important properties of classical differential calculus.

The numerical examples demonstrated that the deformable derivative can be effectively applied to polynomial, exponential, and trigonometric functions. The obtained results verified the validity of the relation

$$D^\alpha f(t) = \beta f(t) + \alpha f'(t),$$

where

$$\beta = 1 - \alpha$$

It was observed that when $\alpha = 1$, the deformable derivative reduces to the ordinary derivative, while for $\alpha = 0$, it becomes the identity operator. Therefore, the deformable derivative acts as an interpolation mechanism between a function and its classical rate of change.

The equilibrium analysis of deformable differential equations revealed that equilibrium points can be determined using procedures similar to those employed in classical dynamical systems. For the system

$$D^\alpha x(t) = -kx(t),$$

the equilibrium point was found to be

$$x^* = 0$$

Further stability analysis showed that the corresponding solutions converge toward the equilibrium point for positive values of k . The solutions exhibited exponential decay, indicating asymptotic stability as well as exponential stability.

The Lyapunov stability analysis confirmed these findings. By selecting the positive definite Lyapunov function

$$V(x) = x^2,$$

the derivative of the Lyapunov function along the trajectories was obtained as

$$\dot{V}(x) = -\frac{2(k+\beta)}{\alpha}x^2$$

Since

$$\dot{V}(x) < 0 \text{ for } x \neq 0,$$

the equilibrium point satisfies the conditions for asymptotic stability according to Lyapunov's direct method.

A significant finding of the research is that the stability behaviour of the deformable differential systems still carries that of the corresponding classical system. In other words, the deformation parameter α affects speed of convergence of the system and the system dynamics, which however do not change the basic structure of the system. Because deformable derivative was introduced to represent a process whose behaviour varies over time in a way that is not fully static or dynamic.

On the whole, results show that concepts the system of deformable derivative. The deformable derivative's uses in modern stability analysis are demonstrated by the numerical and theoretical results that verify the validity of the result.

6. Conclusion:

This paper presented a detailed study of stability theory in the context of the deformable derivative. The fundamental properties of the deformable derivative were introduced and its relationship with classical differentiation was established. It was shown that the deformable derivative provides a smooth transition between a function and its

ordinary derivative through the deformation parameter α .

The concepts were reviewed and adapted to deformable differential systems. Several computation of deformable derivatives and the determination of equilibrium points. The obtained solutions demonstrated that many classical stability principles remain valid within the deformable derivative framework.

The Lyapunov method was employed to analyze the stability of deformable systems, and the results confirmed that stable behavior can be effectively characterized using positive definite Lyapunov functions. The study showed that the deformable derivative preserves the essential structure of stability analysis while providing additional flexibility for modeling generalized dynamical phenomena.

Therefore, the deformable derivative represents a promising extension of classical calculus and offers new opportunities for the analysis of nonlinear systems, control problems, fractional-type models, and generalized differential equations.

In conclusion, the deformable derivative serves as a powerful mathematical framework that successfully connects generalized differentiation with classical stability theory, thereby enriching the theoretical foundation and practical applicability of modern dynamical system analysis.

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